Oppgave 3 2019 konte

a) Finn retningsderivate til $\boxed{f(x,y,z) = xyz + \sin x}$ i punktet $(\pi, 0, 3)$ i retninga $\frac{1}{3}[1, 2, 2] = \vec{u}$

$$\vec{u} = \left[\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right] \qquad \|\vec{u}\| = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2}$$

$$= \sqrt{\frac{1}{9} + \frac{4}{9} + \frac{4}{9}} = \sqrt{1} = \underline{1}$$

$$\nabla f = [yz + \cos x, xz, xy]$$

$$\nabla f(\pi, 0, 3) = [0 + (-1), 3\pi, 0] = [-1, 3\pi, 0]$$

$$D_{\vec{u}} f(\pi, 0, 3) = [-1, 3\pi, 0] \cdot \left[\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right] = -\frac{1}{3} + 2\pi + 0 = \underline{\underline{2\pi - \frac{1}{3}}}$$

b) Finn de stasjonære punktene til $g(x,y) = \cos(x^2+y^2)$ hvor (x,y) ligger inni sirkelen med radius $\frac{5}{6}\pi$ og sentrum i origo

$$\frac{\partial g}{\partial x} = -\sin(x^2+y^2)\cdot 2x = -2x\sin(x^2+y^2)$$

$$u = x^2+y^2 \qquad \frac{\partial u}{\partial x} = 2x$$

$$\frac{\partial g}{\partial y} = -\sin(x^2+y^2)\cdot 2y = -2y\sin(x^2+y^2)$$

$$u = x^2+y^2 \qquad \frac{\partial u}{\partial y} = 2y$$

$$-2x\sin(x^2+y^2) = 0 \quad \Rightarrow \quad x = 0 \quad ✓ \quad \boxed{\sin(x^2+y^2) = 0} \quad ✓ \quad \cancel{-2 = 0}$$

$$-2y\sin(x^2+y^2) = 0$$

$$\sin(x^2+y^2)=0 \qquad x^2+y^2=\boxed{k\cdot\pi} \qquad k\geq 0$$

$$\sin(k\cdot\pi)=0$$

$$x=0$$

$$-2y\sin(y^2)=0$$

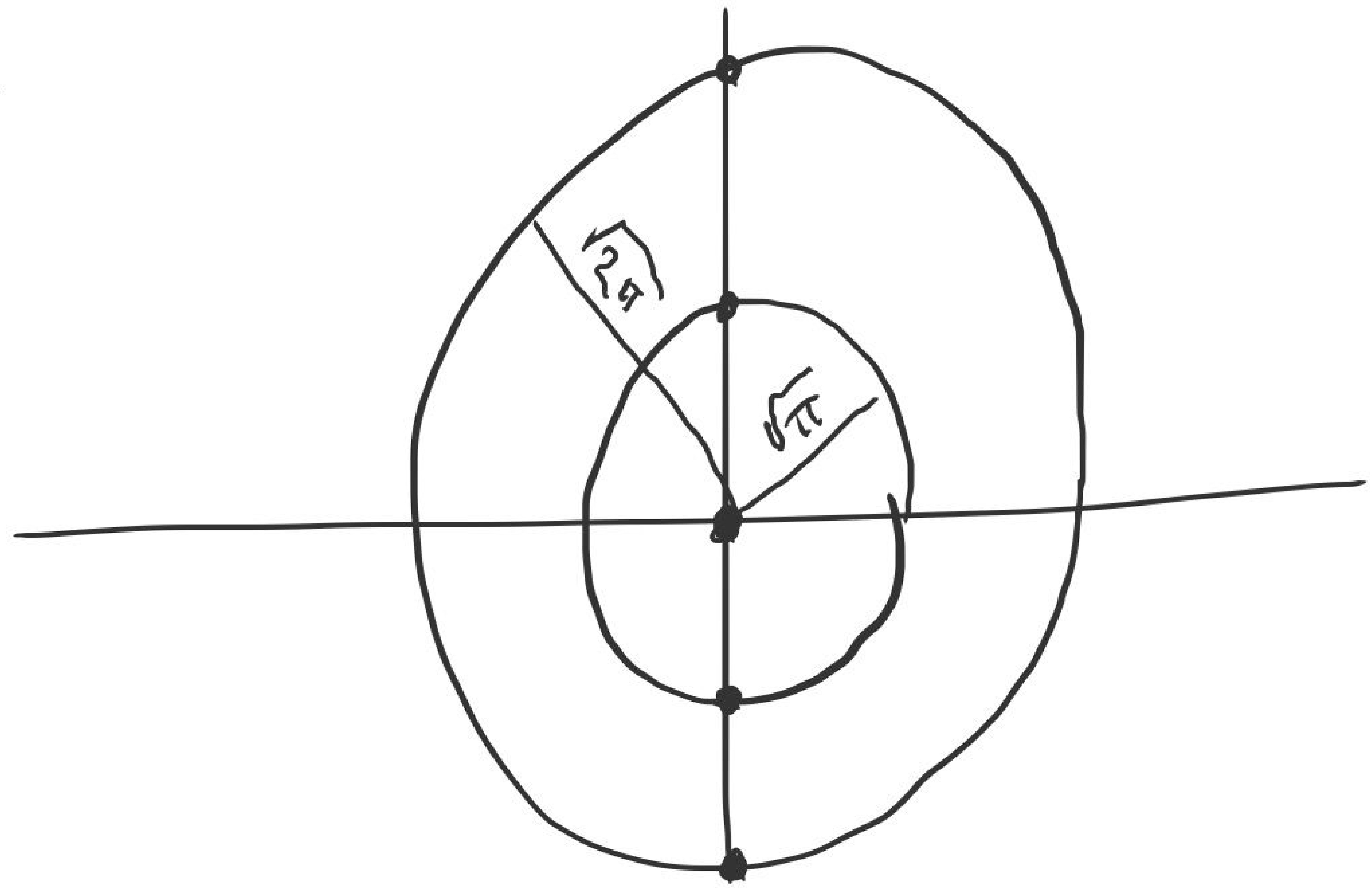
$$y=0$$

$$\sin(y^2)=0$$

$$y^2=k\cdot\pi \qquad k\geq 0$$

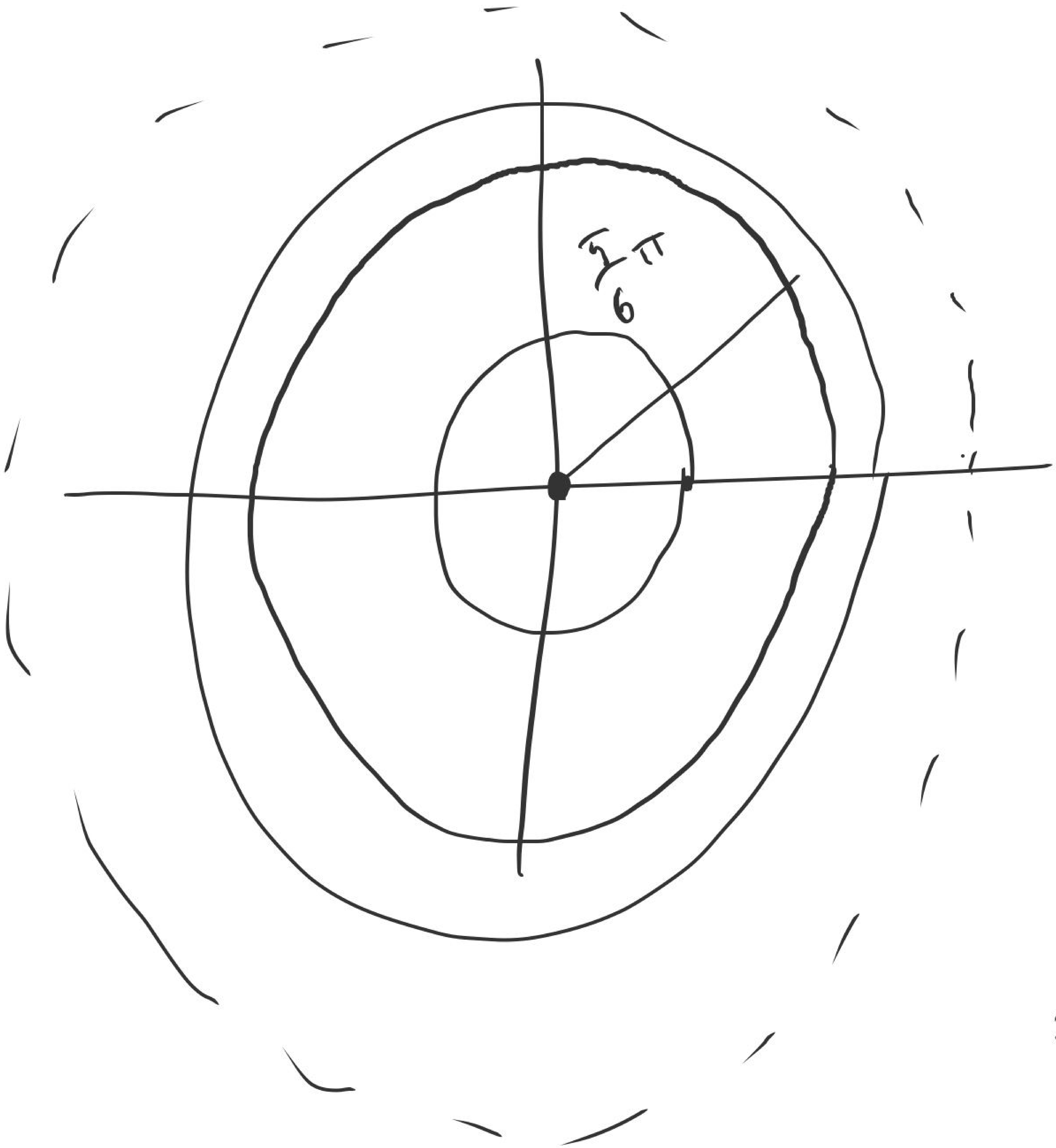
$$y=\pm\sqrt{k\pi}$$

Ingen nye punkter.



$$x^2+y^2=r^2$$

$$x^2+y^2=\boxed{\sqrt{k\pi}}^2$$



$\frac{5}{6}\pi \approx 2.612$

$\sqrt{\pi} = 1.77$

$\sqrt{2\pi} = 2.507$

$\sqrt{3\pi} = 3.07$

Stasjonære pt innafor sirkelen med radius $\frac{5}{6}\pi$ er:

- $(0,0)$
- Sirkelen med radius $\sqrt{\pi}$
- Sirkelen med radius $\sqrt{2\pi}$

$x^2+y^2=0$

$x=0$ og $y=0$

Oppgave 11.3.3.

For hver funksjon

i) Regne ut første og andrederiverte

ii) Finn og klassifiser de kritiske punktene.

a) $f(x,y) = xy^2 + x^2 - 4x$

$f_x = y^2 + 2x - 4$

$f_y = 2xy$

$f_{xx} = 2$ $\quad f_{yx} = 2y$

$f_{xy} = 2y$ $\quad f_{yy} = 2x$

$$Hf = \begin{bmatrix} 2 & 2y \\ 2y & 2x \end{bmatrix}$$

$y^2 + 2x - 4 = 0$

$2xy = 0$

$2 \neq 0 \lor x = 0 \lor y = 0$

$x = 0$

$y^2 - 4 = 0 \quad y = \pm 2$

$(0,2) \quad (0,-2)$

$y = 0$

$2x - 4 = 0$

$x = 2 \quad (2,0)$

$(0,2)$

$(0,-2)$

$(2,0)$

Klassifisere.

$$Hf = \begin{bmatrix} 2 & 2y \\ 2y & 2x \end{bmatrix} \quad i \text{ punktane } (0,2) \quad (2,0) \quad (0,-2)$$

$(0,2)$

$$|Hf| = \begin{vmatrix} 2 & 4 \\ 4 & 0 \end{vmatrix} = -16 < 0 \quad \text{Sadelpunkt.}$$

$(0,-2)$

$$|Hf| = \begin{vmatrix} 2 & -4 \\ -4 & 0 \end{vmatrix} = -16 < 0 \quad \text{Sadelpunkt.}$$

$(2,0)$

$$|Hf| = \begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix} = 8 > 0, \quad 2 > 0 \quad \text{Bunnpunkt.}$$

b)

$f(x,y) = 3x^3 + xy^2 - 9x$

$f_x = 9x^2 + y^2 - 9$

$f_y = 2xy$

$f_{xx} = 18x$

$f_{xy} = 2y$

$f_{yx} = 2y$

$f_{yy} = 2x$

$|Hf| = \begin{vmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{vmatrix}$

$= f_{xx} f_{yy} - f_{xy} f_{yx}$

$9x^2 + y^2 - 9 = 0$

$2xy = 0$

$2xy = 0 \Rightarrow x = 0 \lor y = 0$

$x = 0$

$y^2 - 9 = 0$

$y = \pm 3$

$(0, -3)$

$(0, 3)$

$y = 0$

$9x^2 - 9 = 0$

$9x^2 = 9$

$x^2 = 1$

$x = \pm 1$

$(1, 0)$

$(-1, 0)$

$Hf = \begin{bmatrix} 18x & 2y \\ 2y & 2x \end{bmatrix}$

i puntkand

$(0, -3)$

$(0, 3)$

$(1, 0)$

$(-1, 0)$

$$Hf = \begin{bmatrix} 18x & 2y \\ 2y & 2x \end{bmatrix} \quad ; \quad (0,-3),\ (0,3),\ (-1,0),\ (1,0)$$

Hvis $f_{xx} = 0$ vil $|Hf| \leq 0$

$\underline{(0,-3)}$

$$|Hf| = \begin{vmatrix} 0 & -6 \\ -6 & 0 \end{vmatrix} = -36 < 0 \quad \text{sadelpt.}$$

$\underline{(0,3)}$

$$|Hf| = \begin{vmatrix} 0 & 6 \\ 6 & 0 \end{vmatrix} = -36 < 0 \quad \text{sadelpunkt.}$$

$\underline{(-1,0)}$

$$|Hf| = \begin{vmatrix} -18 & 0 \\ 0 & -2 \end{vmatrix} = 36 > 0 \quad \boxed{-18 < 0} \quad \text{Toppunkt.}$$

$18 > 0$ Bunnpt.

$\underline{(1,0)}$

$$|Hf| = \begin{vmatrix} 18 & 0 \\ 0 & 2 \end{vmatrix} = 36 > 0$$

c)

$f(x,y) = x^3 + 6y^4 + 12xy$

$f_x = 3x^2 + 12y$

$f_y = 24y^3 + 12x$

$f_{xx} = 6x$

$f_{xy} = 12$

$f_{yx} = 12$

$f_{yy} = 72y^2$

$3x^2 + 12y = 0$

$24y^3 + 12x = 0$

$24y^3 + 12x = 0$

$x = -2y^3$

$3(-2y^3)^2 + 12y = 0$

$12y^6 + 12y = 0$

$12y(y^5 + 1) = 0$

$\boxed{y = 0}$

eller

$y^5 = -1 \Rightarrow \boxed{y = -1}$

$(0, 0), (2, -1)$

$$Hf = \begin{bmatrix} 6x & 12 \\ 12 & 72y^2 \end{bmatrix}$$ i punktene $(0,0)$ og $(2,-1)$

<u>$(0,0)$</u>

$$|Hf| = \begin{vmatrix} 0 & 12 \\ 12 & 0 \end{vmatrix} = -144 < 0$$ sadelpt

<u>$(2,-1)$</u>

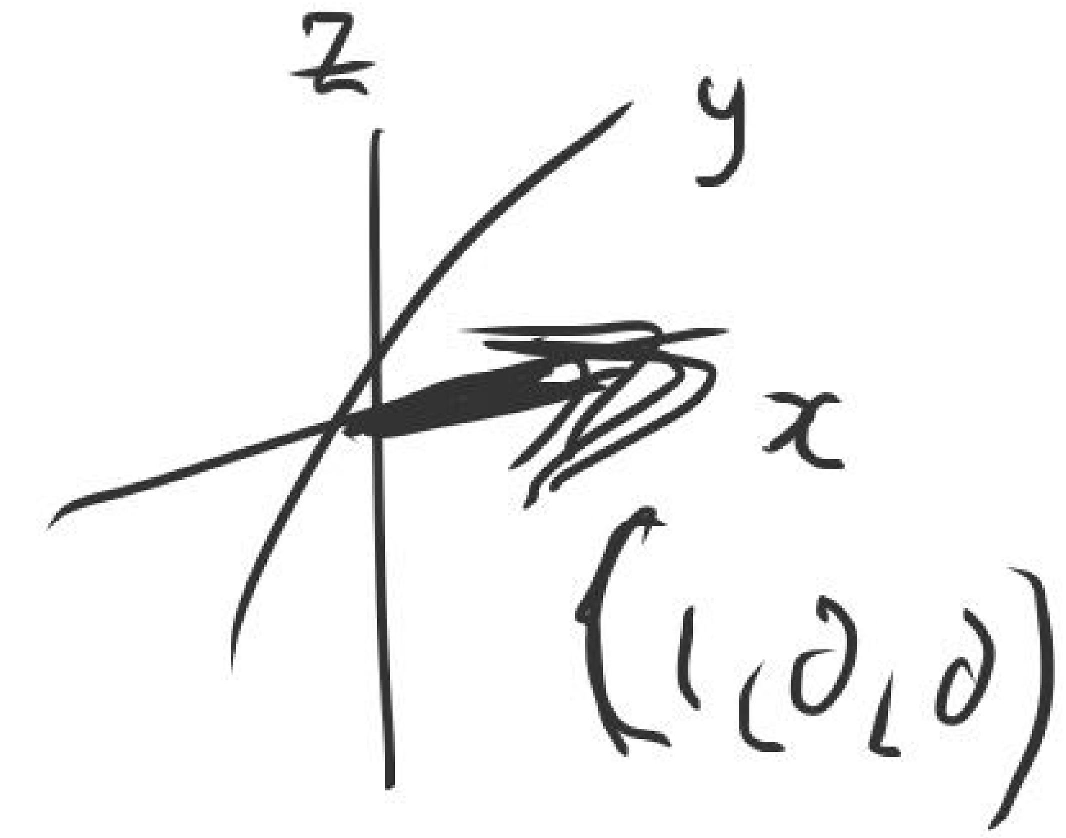
$$|Hf| = \begin{vmatrix} 12 & 12 \\ 12 & 72 \end{vmatrix} = 12 \cdot 72 - 12 \cdot 12 = 720 > 0$$

$12 > 0$ Bunnpunkt.

Eksamen 2013 oppg 3

Gitt $g(x,y,z) = x \cos y + z^2$

a) Finn retningsderiverte i punktet $(0, \pi, 13)$ i positiv x-retning.



$\vec{u} = [1, 0, 0]$

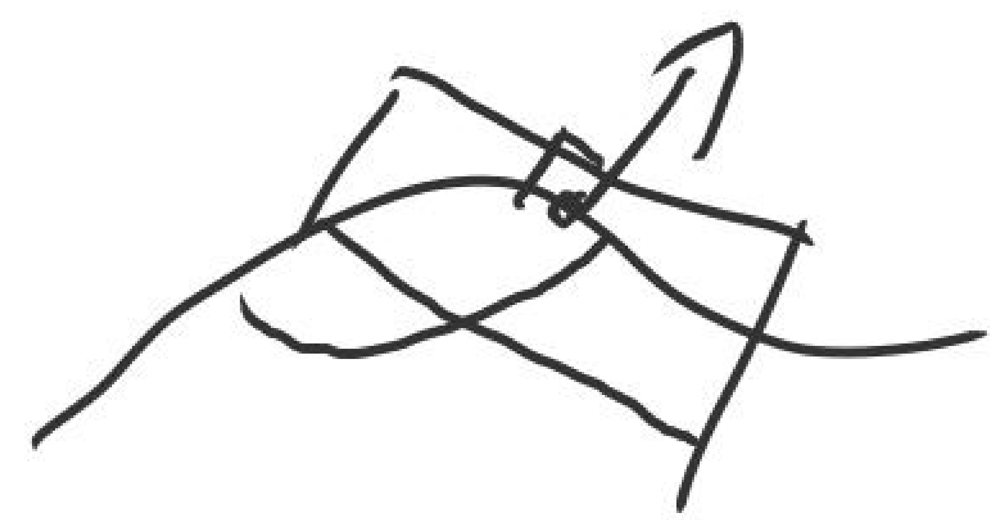
$\nabla g = [\cos y, -x \sin y, 2z]$

$\nabla g(0, \pi, 13) = [-1, 0, 26]$

$D_{\vec{u}} g(0, \pi, 13) = [-1, 0, 26] \cdot [1, 0, 0] = -1 + 0 + 0 = \underline{-1}$

b) Finn tangentplanet til nivå flaten $g(x,y,z) = 25$ i punktet $(0, \frac{\pi}{4}, 5)$.

$$g(0, \frac{\pi}{4}, 5) = 25$$



$$\nabla g = [\cos y, -x \sin y, 2z]$$

$$\nabla g(0, \frac{\pi}{4}, 5) = [\frac{\sqrt{2}}{2}, 0, 10]$$

Formel for plan:

$$\frac{\sqrt{2}}{2}(x-0) + \cancel{0 \cdot (y - \frac{\pi}{4})} + 10(z-5) = 0$$

$$\frac{\sqrt{2}}{2}x + 10z - 50 = 0$$

$$\frac{\sqrt{2}}{2}x + 10z = 50$$

c) Vis at $(0, \frac{\pi}{2}, 0)$ er et kritisk pt.

$$g_x = \cos y$$

$$g_y = -x \sin y$$

$$g_z = 2z$$

$$\boxed{\begin{aligned} g_x(0, \tfrac{\pi}{2}, 0) &= \cos \tfrac{\pi}{2} = 0 \\ g_y(0, \tfrac{\pi}{2}, 0) &= -0 \cdot \sin \tfrac{\pi}{2} = 0 \\ g_z(0, \tfrac{\pi}{2}, 0) &= 2 \cdot 0 = 0 \end{aligned}}$$

Kan du finne alle?

$\cos y = 0 \Rightarrow y = \frac{\pi}{2} + k \cdot \pi$

$\underline{-x \sin y = 0} \Rightarrow x = 0$

$\underline{2z = 0} \Rightarrow z = 0$

$\underline{(0, \frac{\pi}{2} + k\pi, 0)}$ $k = \mathbb{Z}$

$k = \ldots -3, -2, -1,$

$0, 1, 2, 3, \ldots$