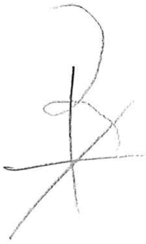
11.2.2

a) $\vec{r}(t) = [\cos t, \sin t, e^{2t}]$



$\vec{v}(t) = [-\sin t, \cos t, 2e^{2t}]$

$v(t) = \|\vec{v}(t)\|$

$= \sqrt{(-\sin t)^2 + (\cos t)^2 + (2e^{2t})^2}$

$= \sqrt{4e^{4t} + 1}$

b) $\vec{r}(t) = [t^2, \cos t + t\sin t, \sin t - t\cos t] \quad t \geq 0$

$\vec{v}(t) = [2t, \cancel{-\sin t} + \cancel{\sin t} + t\cos t, \cancel{\cos t} - (\cancel{\cos t} - t\sin t)]$

$= [2t, t\cos t, t\sin t]$

$v(t) = \|\vec{v}(t)\|$

$= \sqrt{(2t)^2 + (t\cos t)^2 + (t\sin t)^2}$

$= \sqrt{4t^2 + t^2\cos^2 t + t^2\sin^2 t}$

$= \sqrt{t^2(4 + \cos^2 t + \sin^2 t)} = \sqrt{t^2 \cdot 5} = \sqrt{5} \cdot t$

f) I hvilke retningen ut fra (1,2) har f stigningstall lik $\frac{7}{39}$?

$D_{\vec{u}}(f)(a,b) = \nabla f(a,b) \cdot \vec{u}$ hvor $\|\vec{u}\| = 1$.

$\nabla f \cdot \vec{u} = \frac{7}{39}$ $\quad \nabla f = \left[\frac{1}{8}, -\frac{1}{6}\right]$

$u = [x, y]$

$\boxed{\frac{1}{8}x - \frac{1}{6}y = \frac{7}{39}}$ $\quad y = \frac{3}{4}x - \frac{14}{13}$

$\boxed{x^2 + y^2 = 1}$

$$x^2 + \left(\frac{3}{4}x - \frac{14}{13}\right)^2 = 1$$

$$x^2 + \frac{9}{16}x^2 - 2 \cdot \frac{3}{4}x \cdot \frac{14}{13} + \frac{14^2}{13^2} = 1$$

$$\frac{25}{16}x^2 - \frac{84}{52}x + \frac{196}{169} - 1 = 0$$

$$\frac{25}{16}x^2 - \frac{21}{13}x + \frac{27}{169} = 0$$

$x_1 = \frac{12}{13}$

$x_2 = \frac{36}{325}$

$x = \frac{12}{13}$ $\quad y = \frac{3}{4} \cdot \frac{12}{13} - \frac{14}{13} = -\frac{5}{13}$ $\quad \left(\frac{12}{13}, -\frac{5}{13}\right)$

$x = \frac{36}{325}$ $\quad y = \frac{3}{4} \cdot \frac{36}{325} - \frac{14}{13} = -\frac{323}{325}$ $\quad \left(\frac{36}{325}, -\frac{323}{325}\right)$

$\nabla f \cdot \vec{u} = \frac{7}{39}$

$\nabla f = \left[\frac{1}{8}, -\frac{1}{6}\right]$

$\|\vec{u}\| = 1$

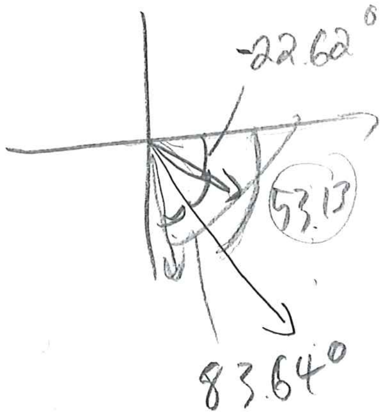
$\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cdot \cos\theta$

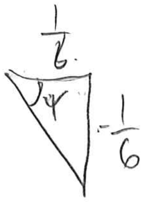
$$\|\nabla f\| = \sqrt{\left(\frac{1}{8}\right)^2 + \left(-\frac{1}{6}\right)^2}$$
$$= \sqrt{\frac{1}{64} + \frac{1}{36}}$$
$$= \sqrt{\frac{25}{576}} = \frac{\sqrt{25}}{\sqrt{576}} = \frac{5}{24}$$

$$\frac{7}{39} = \frac{5}{24} \cdot 1 \cdot \cos\theta$$

$$\cos\theta = \frac{24 \cdot 7}{39 \cdot 5} = \frac{56}{65}$$

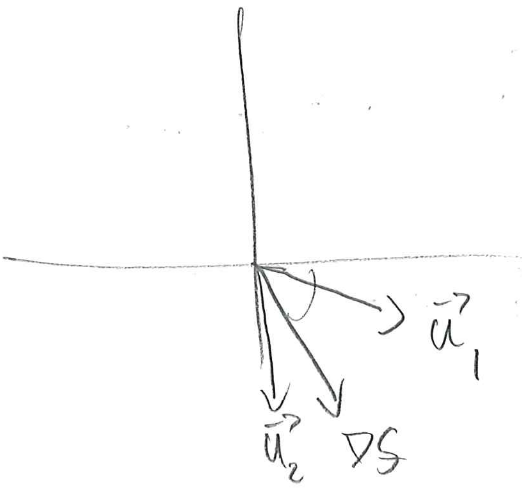
$$\theta = 30.51^\circ$$





$$\tan\psi = \frac{-\frac{1}{6}}{\frac{1}{8}}$$

$$\tan^{-1}\left(-\frac{8}{6}\right) = -53.13^\circ$$



Vinkel mellom ∇f og $\vec{u}$ skal være 30.51°.

Vinkel mellom ∇f og x-aksen er -53.13°

11.2.8

Hvor raskt vokser funksjonen i oppgitt retning, og hvor stor prosent er dette i forhold til maksimal vekst?

a) $f(x,y) = x \cdot y$, punktet $(x,y) = (1,1)$

retningen $[7,-1]$.

$\nabla f = [y, x]$. $\nabla f(1,1) = [1,1]$.

Maksimal vekst er derfor $\|\nabla f(1,1)\| = \sqrt{1^2+1^2} = \underline{\sqrt{2}}$

$D_{\vec{u}} f = \nabla f \cdot \vec{u}$ $\qquad \|\vec{u}\| = 1$.

$\|[7,-1]\| = \sqrt{49+1} = \boxed{\sqrt{50}} \neq 1$.

$\vec{u} = \left[\frac{7}{\sqrt{50}}, \frac{-1}{\sqrt{50}}\right]$

$D_{\vec{u}} f = [1,1] \cdot \left[\frac{7}{\sqrt{50}} \; \frac{-1}{\sqrt{50}}\right]$

$= \frac{6}{\sqrt{50}} = \frac{6}{5 \cdot \sqrt{2}}$ Hvor raskt funksjonen vokser.

Prosent: $\frac{\left(\frac{6}{5\cdot\sqrt{2}}\right)}{\sqrt{2}} = \frac{6}{5\cdot\sqrt{2}\cdot\sqrt{2}} = \frac{6}{10} = 0.6 = 60\%$

b) $f(x,y) = \frac{1}{12}xy^3$ punktet $(x,y) = (\frac{1}{2}, 2)$

retningen $[24, -7]$.

$$\nabla f = \left[\frac{1}{12}y^3, \frac{1}{4}xy^2\right]$$

$$\nabla f(\tfrac{1}{2}, 2) = \left[\frac{2}{3}, \frac{1}{2}\right]$$

$$\|\nabla f(\tfrac{1}{2}, 2)\| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{4}{9} + \frac{1}{4}} = \sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$$

$$\|[24, -7]\| = \sqrt{24^2 + (-7)^2} = \sqrt{576 + 49} = \sqrt{625} = 25$$

$$\vec{u} = \left[\frac{24}{25}, \frac{-7}{25}\right]$$

$$D_{\vec{u}}f = \left[\frac{2}{3}, \frac{1}{2}\right] \cdot \left[\frac{24}{25}, \frac{-7}{25}\right] = \frac{48}{75} - \frac{7}{50} = \underline{\underline{\frac{1}{2}}}$$

I prosent: $\frac{\frac{1}{2}}{\frac{5}{6}} = \frac{1}{2} \cdot \frac{6}{5} = \frac{6}{10} = 0.6 = 60\%$

c) $g(x,y) = 3x^2 + y^2 + 4y$ i punktet $(3,-5)$
i retningen $[3,1]$.

$\nabla g = [6x, 2y+4]$ $\qquad$ $\nabla g(3,-5) = [18,-6]$

$\|\nabla g(3,-5)\| = \sqrt{18^2 + (-6)^2}$

$= \sqrt{324+36}$

$= \sqrt{360} = \sqrt{36 \cdot 10}$

$= \sqrt{36} \cdot \sqrt{10} = \underline{6\sqrt{10}}$

$\|[3,1]\| = \sqrt{3^2+1^2} = \sqrt{10}$

$\vec{u} = \left[\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right]$

$D_{\vec{u}}g = [18,-6] \cdot \left[\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right]$

$= \frac{54}{\sqrt{10}} - \frac{6}{\sqrt{10}} = \frac{48 \cdot \sqrt{10}}{\sqrt{10} \cdot \sqrt{10}}$

$= \frac{48 \cdot \sqrt{10}}{10} = 4.8 \cdot \sqrt{10}$

I prosent $\frac{4.8 \cdot \cancel{\sqrt{10}}}{6 \cdot \cancel{\sqrt{10}}} = \frac{4.8}{6} = \frac{48}{60} = \frac{12}{15} = \frac{4}{5} = \underline{\underline{80\%}}$

d)

$g(x,y) = 4x^2 + y^2 + 12y$ punktet $(1,-3)$ retningen $[4,3]$.

$\nabla g = [8x, 2y+12]$ $\nabla g(1,-3) = [8,6]$

$\|\nabla g(1,-3)\| = \sqrt{64+36} = \sqrt{100} = 10$

$\|[4,3]\| = \sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25} = 5$ $\vec{u} = [\frac{4}{5}, \frac{3}{5}]$

$D_{\vec{u}} g = [8,6] \cdot [\frac{4}{5}, \frac{3}{5}] = \frac{32}{5} + \frac{18}{5} = \frac{50}{5} = \underline{\underline{10}}$

I Prosent: $\frac{10}{10} = 1 = 100\%$

Kunne også sett at $[4,3] = \frac{1}{2} \cdot \nabla g$, så peker i retningen med størst vekst.