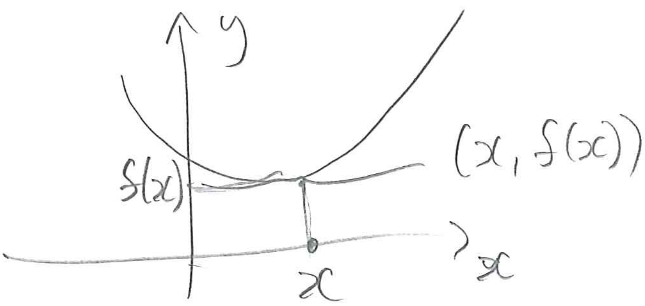
Funksjoner med flere variable.

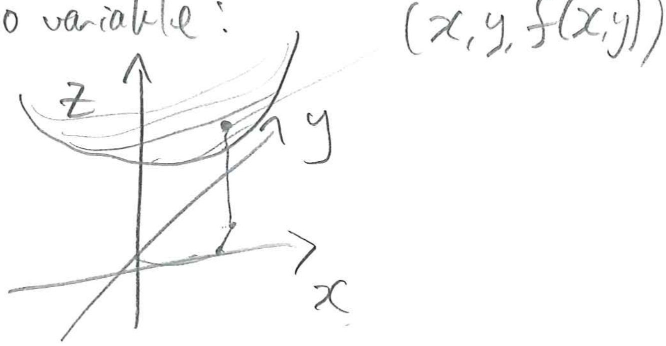
$$f(x,y) = 3x^2 - 2xy + 1$$

$$g(x,y,z) = xyz$$

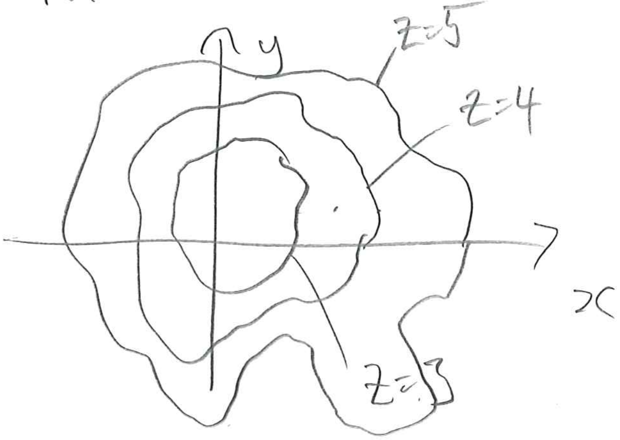
Én variabel:



To variable:

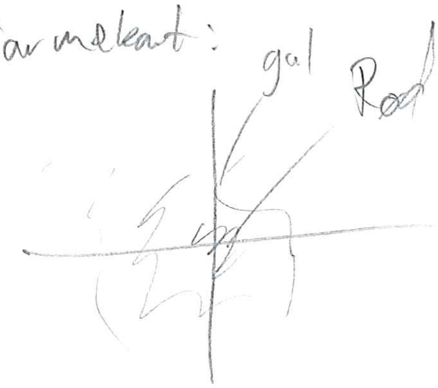


Nivåkurver:



Som høgdekurver på kart.

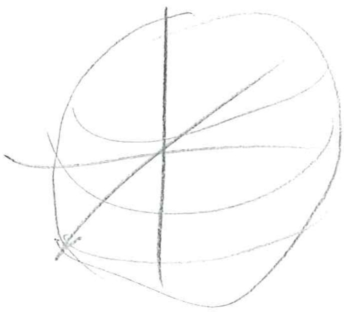
Varmekart:



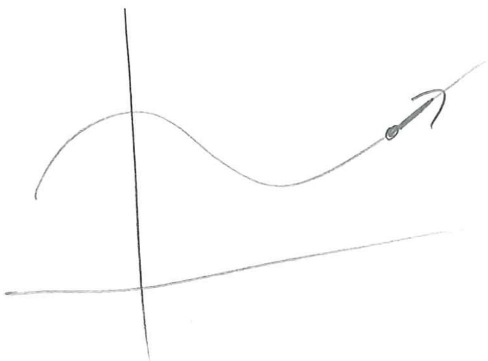
Tre variable:

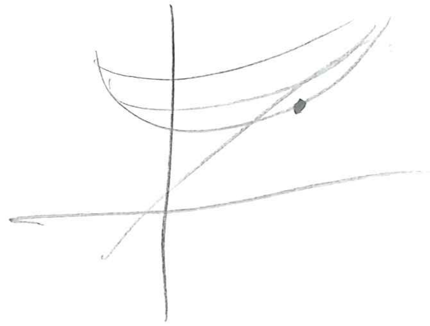
Trenger fire dimensjoner (3 inn, 1 ut).

Kan tegne nivå flater



Vi vil <u>derivere</u> disse.





Vil finne deriverte <u>langs</u> x-aksen og <u>langs</u> y-aksen.

Partiellt derivatete.

Skrivemåter

$$\frac{\partial f}{\partial x} = f_x \qquad \frac{df}{dt}$$

$$\frac{\partial f}{\partial y} = f_y \qquad \left(\frac{\partial f}{\partial z} = f_z \right)$$

Hvordan regner vi ut dette?

Regner $\frac{\partial f}{\partial x}$ ved å late som y (og z) er konstanter.

<u>Eks:</u>

$$f(x,y) = 3x^2 + \boxed{4xy} - y^3 + 2$$

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= 6x + 4y \\ \frac{\partial f}{\partial y} &= 4x - 3y^2 \end{aligned}\right\}$$ Formel for stigning om vi går langs x- eller y-aksen.

$$\frac{\partial}{\partial x}(4xy) = \frac{\partial}{\partial x}(4y \cdot x) = 4y \frac{\partial}{\partial x}(x) = 4y \cdot 1 = 4y$$

$f(x,y) = x^2 y^2$

$\frac{\partial f}{\partial x} = 2xy^2$ $\qquad$ $\frac{\partial f}{\partial y} = 2x^2 y$

$(\sin(kx))' = k\cos kx$

$f(x,y) = \sin(xy)$

$\frac{\partial f}{\partial x} = y\cos(xy)$

$\frac{\partial f}{\partial y} = x\cos(xy)$

$f(x,y) = \sin x + \cos y$

$\frac{\partial f}{\partial x} = \cos x$

$\frac{\partial f}{\partial y} = -\sin y$

Dobbelderivasjon:

Fire typer:

Deriver mhp x, så mhp x igjen.

— " — x, — " — y

— " — y, — " — x

— " — y, — " — y

$$\frac{\partial}{\partial x}\frac{\partial}{\partial x} f = \frac{\partial^2 f}{(\partial x)^2} = f_{xx}$$

$$\frac{\partial}{\partial y}\frac{\partial}{\partial x} f = \frac{\partial^2 f}{\partial y \partial x} = f_{xy}$$

$$\frac{\partial}{\partial x}\frac{\partial}{\partial y} f = \frac{\partial^2 f}{\partial x \partial y} = f_{yx}$$

For alle „pene" funksjoner er disse <u>like</u>.

$$\frac{\partial}{\partial y}\frac{\partial}{\partial y} f = \frac{\partial^2 f}{(\partial y)^2} = f_{yy}$$

$$f(x,y) = 3x^2 + 4xy - y^3 + 2$$

$$\frac{\partial f}{\partial x} = 6x + 4y \qquad \frac{\partial f}{\partial y} = 4x - 3y^2$$

$$\frac{\partial^2 f}{(\partial x)^2} = 6 \qquad \frac{\partial^2 f}{(\partial y)^2} = -6y$$

$$\frac{\partial^2 f}{\partial y \partial x} = 4 \quad = \quad \frac{\partial^2 f}{\partial x \partial y} = 4$$

$$f(x,y) = x^2 y^2$$

$$\frac{\partial f}{\partial x} = 2xy^2 \qquad \frac{\partial f}{\partial y} = 2x^2 y$$

$$\frac{\partial^2 f}{(\partial x)^2} = 2y^2 \qquad \frac{\partial^2 f}{(\partial y)^2} = 2x^2$$

$$\frac{\partial^2 f}{\partial y \partial x} = 4xy \quad = \quad \frac{\partial^2 f}{\partial x \partial y} = 4xy$$

Er like dersom både $\frac{\partial^2 f}{\partial x \partial y}$ og $\frac{\partial^2 f}{\partial y \partial x}$ er kontinuerlige.

Eksempel, wikipedia:

$$f(x,y) = \begin{cases} \frac{xy(x^2 - y^2)}{x^2 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Gradient til en funksjon

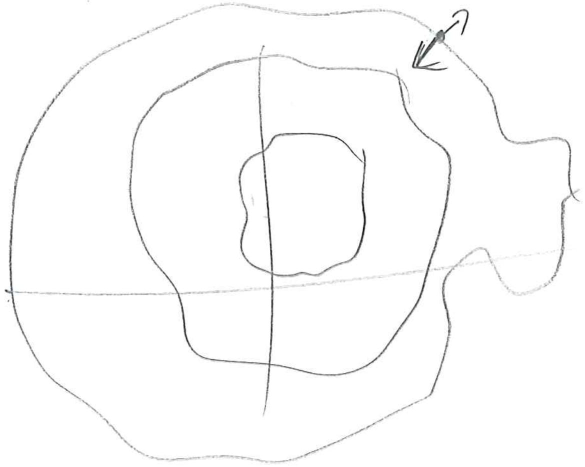
$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) \qquad \left(\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)\right)$$

Har en veldig viktig egenskap:

- Vektoren ∇f peker i den retninga hvor funksjonen vokser mest.
- Lengden $|\nabla f|$ er hvor fort funksjonen vokser i den retninga.

Tredje bonus:

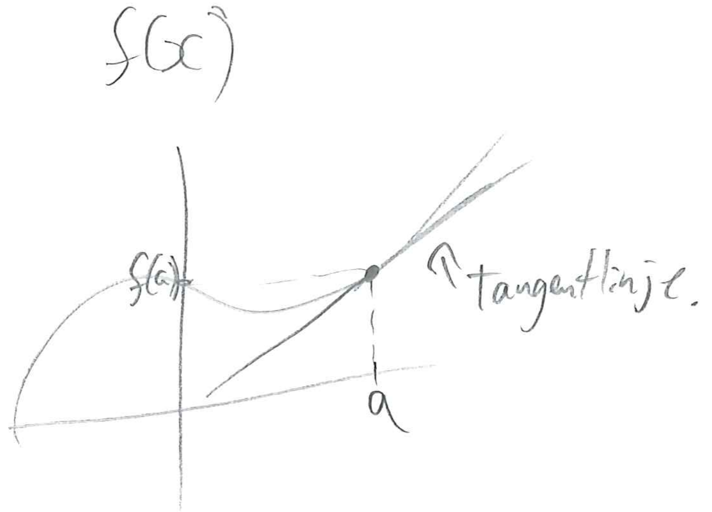
Gradienten peker 90° på nivålinjer.

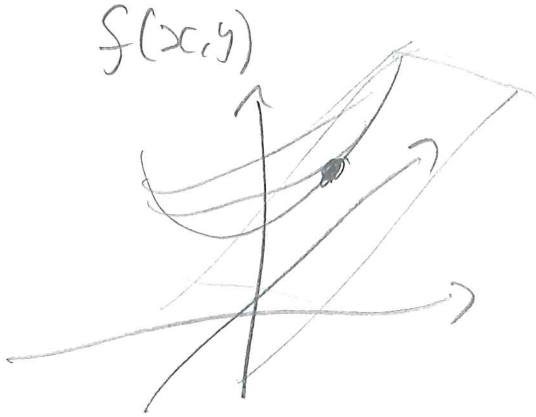


Retningsderiverte: Vi velger en „retning" dvs en vektor $\vec{u}$ med lengde 1.

Den er den retningsderiverte i (a,b) i retning $\vec{u}$ gitt ved

$$D_{\vec{u}} f(a,b) = \nabla f(a,b) \cdot \vec{u}$$





Formel for tangentlinje gjennom $(a, f(a))$

$$y = f(a) + f'(a) \cdot (x-a)$$

Formel for tangentplan gjennom $(a, b, f(a,b))$

$$z = f(a,b) + \nabla f(a,b) \cdot \begin{pmatrix} x-a \\ y-b \end{pmatrix}$$

Eks: $f(x,y) = 3x^2 + 4xy - y^3 + 2$

$$f_x = 6x + 4y \qquad f_y = 4x - 3y^2$$

$$\nabla f = (6x+4y,\ 4x-3y^2)$$

Se på punktet $(1,1)$. $\quad \nabla f(1,1) = (10, 1)$

Funksjonen vokser rasktest i retning $(10,1)$ fra punktet $(1,1)$.

Formel for tangent plan i $(1,1,f(1,1))$ er:

$$\boxed{\begin{aligned} f(1,1) &= 8 \\ \nabla f(1,1) &= (10,1) \end{aligned}}$$

$$z = 8 + (10,1)\cdot\begin{pmatrix} x-1 \\ y-1 \end{pmatrix}$$

$$= 8 + 10(x-1) + 1\cdot(y-1)$$

$$= 8 + 10x - 10 + y - 1$$

$$z = 10x + y - 3$$

$$3 = 10x + y - z$$

Punktet $x = -0.5$, $y = 0.5$.

$$f(-0.5, 0.5) = \frac{3}{4} - 1 - \frac{1}{8} + 2 = \frac{13}{8}$$

$$\nabla f(-0.5, 0.5) = \left(-1, -2 - 3\cdot\frac{1}{4}\right) = \left(-1, -\frac{11}{4}\right)$$

$$z = \frac{13}{8} + \left(-1, -\frac{11}{4}\right)\cdot\begin{pmatrix} x+\frac{1}{2} \\ y-\frac{1}{2} \end{pmatrix} = \frac{13}{8} - \left(x+\frac{1}{2}\right) - \frac{11}{4}\left(y-\frac{1}{2}\right)$$

$$= \frac{13}{8} - x - \frac{1}{2} - \frac{11}{4}y + \frac{11}{8} = -x - \frac{11}{4}y + \boxed{\frac{20}{8}} \quad \frac{5}{2}$$

$$\frac{5}{2} = x + \frac{11}{4}y + z$$