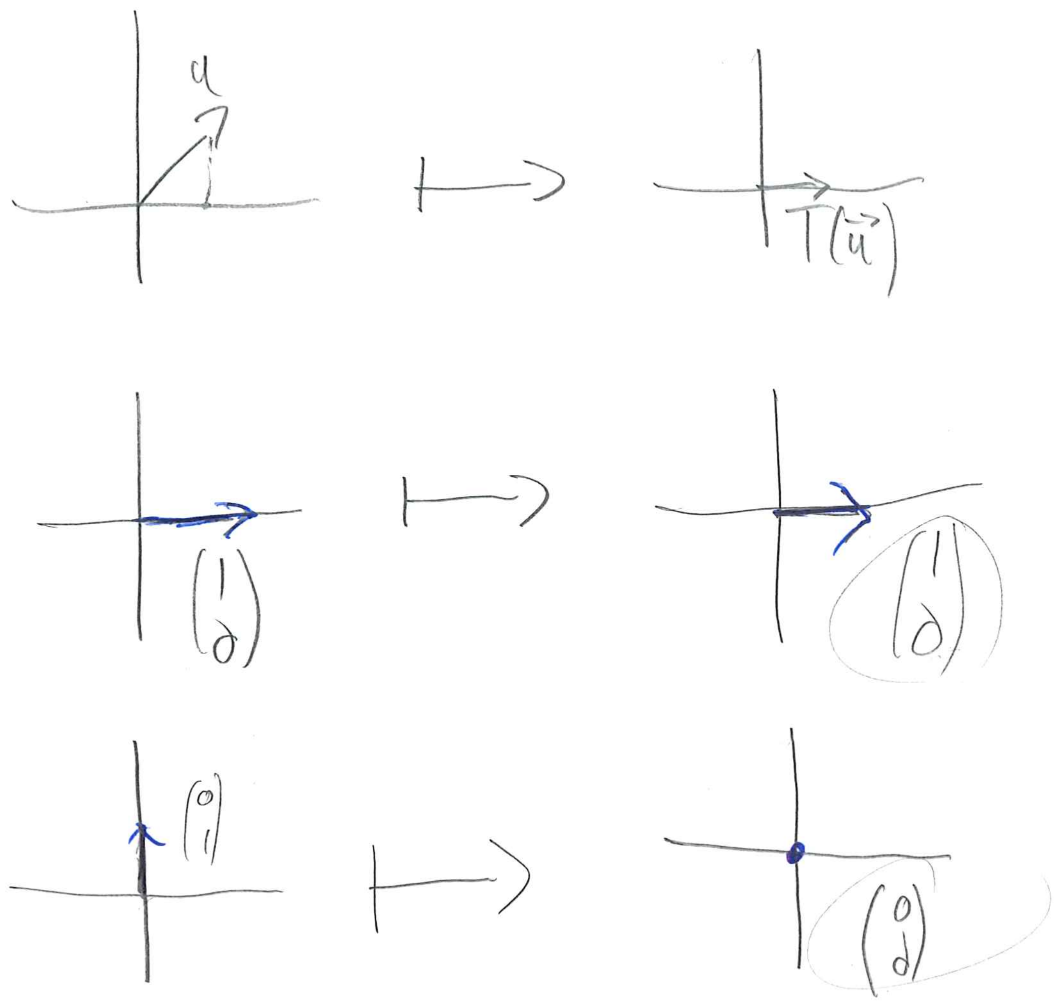
Oppgaver

9.5.2 Finn standardmatrisen til:

a) $T: \mathbb{R}^2 \to \mathbb{R}^2$ definert ved at

$T(\vec{u})$ er projeksjonen på x-aksen.

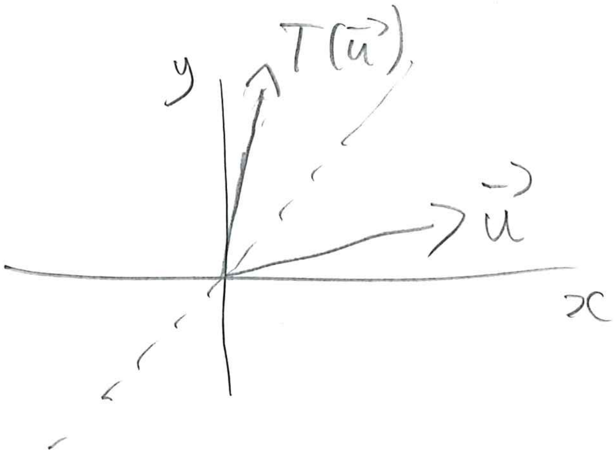


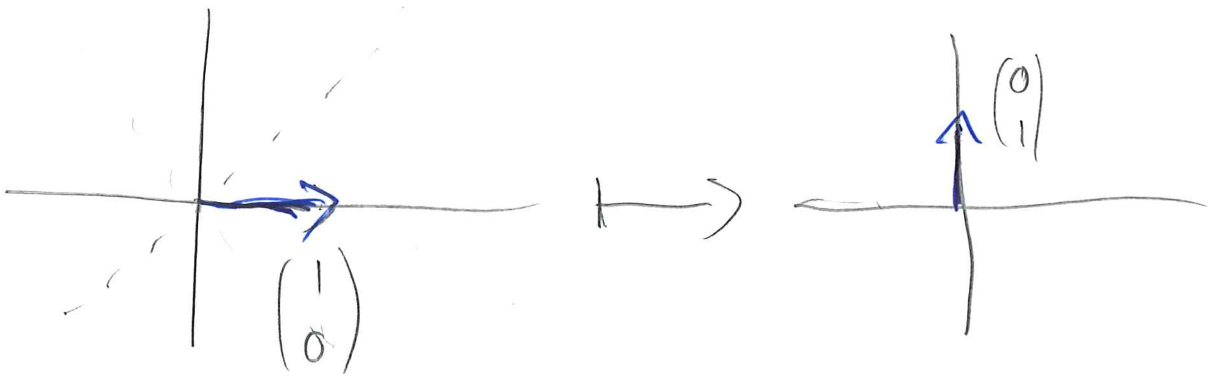
$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

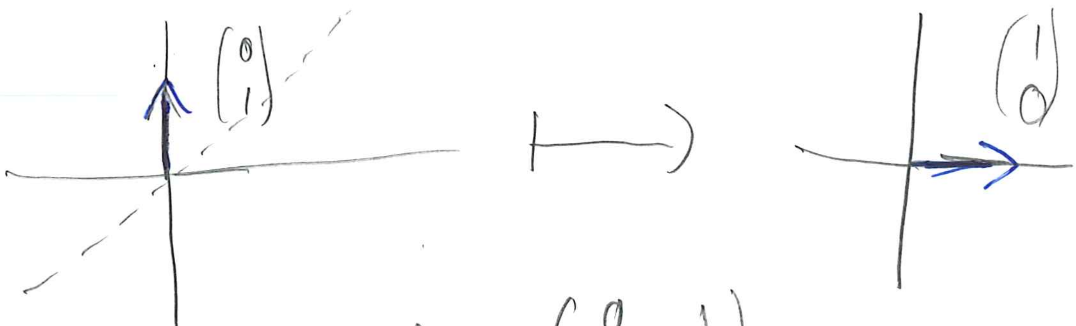
$$T\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\0\end{pmatrix}$$

$$\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\0\end{pmatrix}$$

b) $T:\mathbb{R}^2\to\mathbb{R}^2$ definert ved $T(\vec{u})$ er refleksjonen av $\vec{u}$ om linja $x=y$





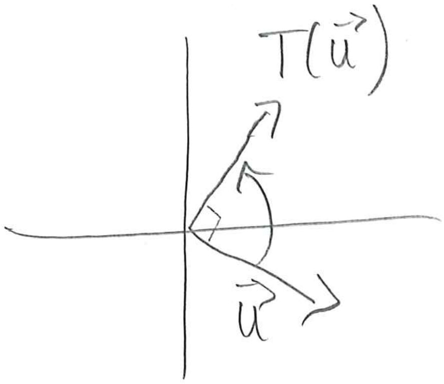


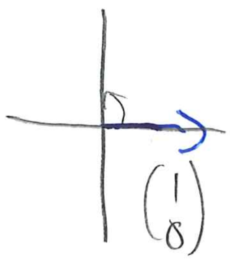
$$A=\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

c) $T: \mathbb{R}^2 \to \mathbb{R}^2$ definert ved

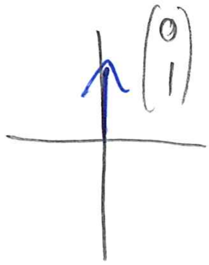
$T(\vec{u})$ er rotasjon av $\vec{u}$

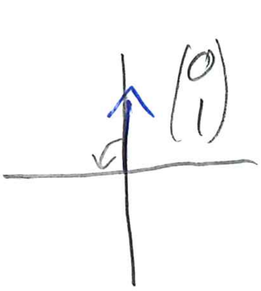
90° mot klokka om origo.



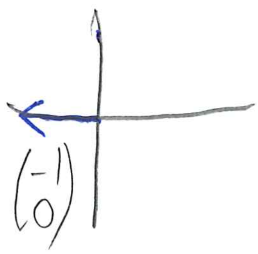


$\longmapsto$



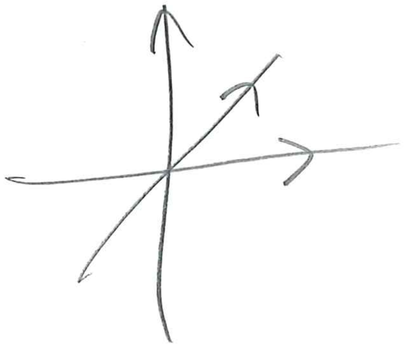


$\longmapsto$



$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$



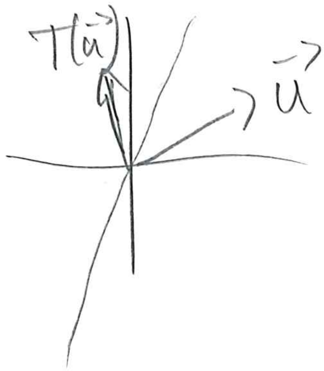
$$\begin{pmatrix}1\\0\\0\end{pmatrix} \longmapsto \begin{pmatrix}v_1\end{pmatrix}$$

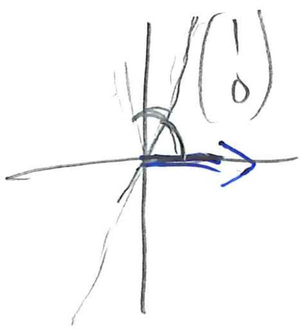
$$\begin{pmatrix}0\\1\\0\end{pmatrix} \longmapsto \begin{pmatrix}v_2\end{pmatrix} \qquad A = \begin{pmatrix}v_1 & v_2 & v_3\end{pmatrix}$$

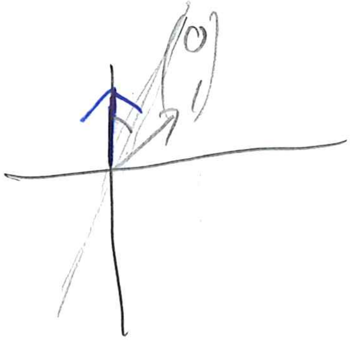
$$\begin{pmatrix}0\\0\\1\end{pmatrix} \longmapsto \begin{pmatrix}v_3\end{pmatrix}$$

<u>Ekstra:</u> $T: \mathbb{R}^2 \to \mathbb{R}^2$

$T(\vec{u})$ er refleksjon langs linja $y = 2x$







$\tan\theta = 2$

$\theta = 63.43°$

9.5.3 | Avgjør om funksjonene er lineær transformasjoner.

Krav:

- $T(c\cdot\vec{u}) = c\cdot T(\vec{u})$

- $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$

a) $T\ell : \mathbb{R}^3 \to \mathbb{R}$ definert ved

$$T(\vec{u}) = \vec{u}\cdot\vec{a}, \quad \text{med } \vec{a} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

Vis krav 1: La $\vec{u} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ og c være et tall.

$$T(c\cdot\vec{u}) = T\left(c\cdot\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = T\left(\begin{pmatrix} cx \\ cy \\ cz \end{pmatrix}\right)$$

$$= \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}\cdot\begin{pmatrix} c\cdot x \\ cy \\ cz \end{pmatrix} = 2cx - 2cy + cz$$

$$= c\cdot(2x - 2y + z) = c\cdot\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}\cdot\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$= c\cdot T(\vec{u})$$

La $\vec{u} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ og $\vec{v} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$

$$T(\vec{u}+\vec{v}) = T\left(\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}\right)$$

$$= T\left(\begin{pmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{pmatrix}\right)$$

$$= \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{pmatrix}$$

$$= 2(x_1+x_2) - 2(y_1+y_2) + (z_1+z_2)$$

$$= 2x_1 - 2y_1 + z_1 + 2x_2 - 2y_2 + z_2$$

$$= \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

$$= T(\vec{u}) + T(\vec{v})$$

Krav 2 holder også.

Bedre måte:

Alle lineærtransformasjoner kan skrives som

$$T(\vec{u}) = A \cdot \vec{u}$$

for en matrise A. Prøv å finn denne matrisen.

$$T(\vec{u}) = \vec{u} \cdot \vec{a} = \vec{a} \cdot \vec{u} = (\vec{a})^T \cdot \vec{u}$$

$$= \begin{pmatrix} 2 & -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$= A \cdot \vec{u} \quad \text{med } A = (\vec{a})^T$$

b) Protip: Hvis T er en lineærtransformasjon, så må $T(\vec{0}) = \vec{0}$

Tester: $T(\vec{u}) = \vec{u} + \vec{a}$ hvor $\vec{a} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$$T(\vec{0}) = \vec{0} + \vec{a} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Så T kan ikke være en lineærtransformasjon.

c) $T: \mathbb{R}^2 \to \mathbb{R}^2$ er def ved

$$T\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} x^2 + y^2 \\ xy \end{pmatrix}$$

$$T\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 0^2 + 0^2 \\ 0 \cdot 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Kan være lineær transformasjon fremdeles.

Er $T\left(2 \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) = 2 \cdot T\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)$?

$$T\left(\begin{pmatrix} 2 \\ 2 \end{pmatrix}\right) = \begin{pmatrix} 2^2 + 2^2 \\ 2 \cdot 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} \neq$$

$$2 \cdot T\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) = 2 \cdot \begin{pmatrix} 1^2 + 1^2 \\ 1 \cdot 1 \end{pmatrix} = 2 \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

T tilfredsstiller <u>ikke</u> krav 1.

Hvis vi tror funksjonen ikke er en lineær transformasjon:

Finn et moteksempel på en av kravene.

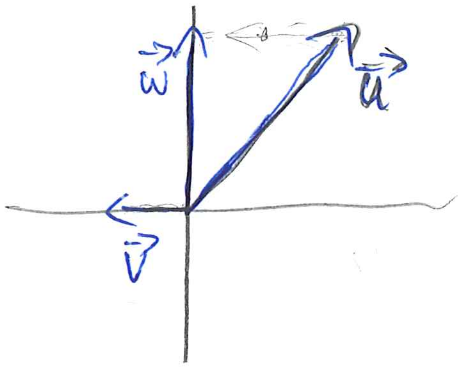
Hvis den er en lineær transformasjon:

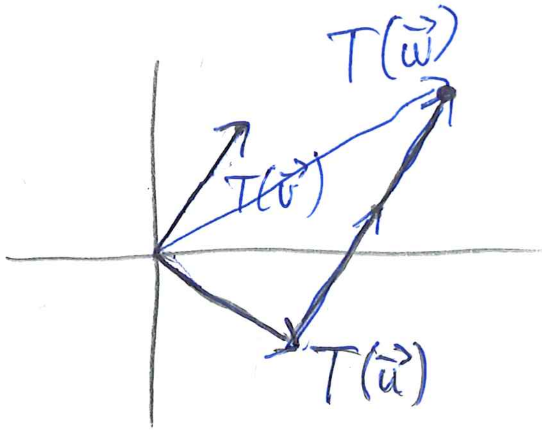
Én av disse.
1. Sjekk/bevis kravene. } Sjekk begge
2. Finn en matrise A slik at

$$T(\vec{u}) = A \cdot \vec{u}.$$

Linear Algebra

1.8.18 Tegn $T(\vec{w})$ på høyre tegning.





① Skriv $\vec{w}$ som en kombinasjon av $\vec{u}$ og $\vec{v}$.

$$\vec{w} = \vec{u} + 2 \cdot \vec{v}$$

② $$T(\vec{w}) = T(\vec{u} + 2\vec{v}) = T(\vec{u}) + T(2 \cdot \vec{v})$$
$$= T(\vec{u}) + 2 \cdot T(\vec{v})$$

③ Tegn $T(\vec{u}) + 2T(\vec{v})$.

1.8.19 La $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ og $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\vec{y}_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ og $\vec{y}_2 = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$,

og $T(\vec{e}_1) = \vec{y}_1$ og $T(\vec{e}_2) = \vec{y}_2$

Finn $T\left(\begin{bmatrix} 5 \\ -3 \end{bmatrix}\right)$ og $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)$

① Skriv $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$ som kombinasjon av $\vec{e}_1$ og $\vec{e}_2$

$$\begin{bmatrix} 5 \\ -3 \end{bmatrix} = 5 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 3 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

② $$T\left(\begin{bmatrix} 5 \\ -3 \end{bmatrix}\right) = 5 \cdot T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) - 3 \cdot T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

$$= 5 \cdot \begin{bmatrix} 2 \\ 5 \end{bmatrix} - 3 \cdot \begin{bmatrix} -1 \\ 6 \end{bmatrix} = \begin{bmatrix} 10 \\ 25 \end{bmatrix} + \begin{bmatrix} 3 \\ -18 \end{bmatrix}$$

$$= \begin{bmatrix} 13 \\ 7 \end{bmatrix}$$

---

① Skriv $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ som kombinasjon av $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ og $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

② $$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = T\left(x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

$$= x_1 \cdot T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) + x_2 \cdot T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

$$= x_1 \cdot \begin{bmatrix} 2 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 6 \end{bmatrix} = \begin{bmatrix} 2x_1 - x_2 \\ 5x_1 + 6x_2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

9.5.4

Finn standardmatrisen til T når:

a) $T\left(\begin{pmatrix}1\\0\end{pmatrix}\right)=\begin{pmatrix}3\\-2\end{pmatrix}$ $\quad T\left(\begin{pmatrix}1\\1\end{pmatrix}\right)=\begin{pmatrix}4\\-1\end{pmatrix}$.

$$T\left(\begin{pmatrix}1\\0\end{pmatrix}\right)=\begin{pmatrix}3\\-2\end{pmatrix}$$

$$T\left(\begin{pmatrix}0\\1\end{pmatrix}\right)=T\left(\begin{pmatrix}1\\1\end{pmatrix}-\begin{pmatrix}1\\0\end{pmatrix}\right)=T\left(\begin{pmatrix}1\\1\end{pmatrix}\right)-T\left(\begin{pmatrix}1\\0\end{pmatrix}\right)$$

$$=\begin{pmatrix}4\\-1\end{pmatrix}-\begin{pmatrix}3\\-2\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}$$

$$\underline{A=\begin{pmatrix}3&1\\-2&1\end{pmatrix}}$$

b) $T\left(\begin{pmatrix}3\\4\end{pmatrix}\right)=\begin{pmatrix}25\\0\end{pmatrix}$ $\quad T\left(\begin{pmatrix}-4\\3\end{pmatrix}\right)=\begin{pmatrix}0\\25\end{pmatrix}$

$T\left(\begin{pmatrix}1\\0\end{pmatrix}\right)=???$ $\quad T\left(\begin{pmatrix}0\\1\end{pmatrix}\right)=???$

$$\begin{pmatrix}1\\0\end{pmatrix}=a\cdot\begin{pmatrix}3\\4\end{pmatrix}+b\cdot\begin{pmatrix}-4\\3\end{pmatrix}$$

$b=-\frac{4}{3}a$

$3a-4b=1$

$4a+3b=0$

$3a-4\left(-\frac{4}{3}a\right)=1$

$3a+\frac{16}{3}a=1$

$9a+16a=3$

$a=3/25$

$a = \frac{3}{25}$ $\qquad b = -\frac{4}{\cancel{3}} \cdot \frac{\cancel{3}}{25} = -\frac{4}{25}$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{3}{25}\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \frac{4}{25}\begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

$$T\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) = T\left(\frac{3}{25}\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \frac{4}{25}\begin{pmatrix} -4 \\ 3 \end{pmatrix}\right)$$

$$= \frac{3}{25} T\left(\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right) - \frac{4}{25} T\left(\begin{pmatrix} -4 \\ 3 \end{pmatrix}\right)$$

$$= \frac{3}{25} \cdot \begin{pmatrix} 25 \\ 0 \end{pmatrix} - \frac{4}{25} \cdot \begin{pmatrix} 0 \\ 25 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$T\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = ???$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = a\begin{pmatrix} 3 \\ 4 \end{pmatrix} + b \cdot \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

$3a - 4b = 0$

$4a + 3b = 1$

$a = \frac{4}{3} b$

$\frac{16}{3} b + 3b = 1$

$16b + 9b = 3$

$b = \frac{3}{25}$

$a = \frac{4}{3} \cdot \frac{3}{25} = \frac{4}{25}$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{4}{25}\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \frac{3}{25}\begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

$$T\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = \frac{4}{25} \cdot T\left(\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right) + \frac{3}{25} \cdot T\left(\begin{pmatrix} -4 \\ 3 \end{pmatrix}\right)$$

$$= \frac{4}{25} \cdot \begin{pmatrix} 25 \\ 0 \end{pmatrix} + \frac{3}{25} \cdot \begin{pmatrix} 0 \\ 25 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

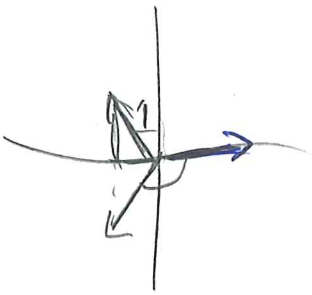
$$A = \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$$

1.9.7 Finn standardmatrisen til T

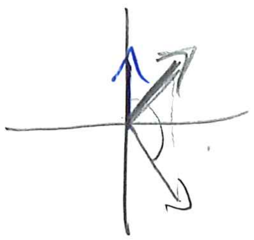
når T: 135°

Først roterer $-\frac{3\pi}{4}$ radianer (med klokka),

så reflekterer gjennom x-aksen.



$$\longrightarrow \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$



$$\longrightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$A = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$