2.1.3

$$A = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix} \qquad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Regn ut

$3I_2 - A$ og $(3I_2)\cdot A$

$$3I_2 = 3\cdot\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3\cdot 1 & 3\cdot 0 \\ 3\cdot 0 & 3\cdot 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$3I_2 - A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 3-4 & 0-(-1) \\ 0-5 & 3-(-2) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -5 & 5 \end{bmatrix}$$

$$(3I_2)\cdot A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\cdot\begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix}$$

$\parallel$

$$3\cdot(I_2\cdot A) = \begin{bmatrix} 3\cdot 4 + 0\cdot 5 & 3\cdot(-1) + 0\cdot(-2) \\ 0\cdot 4 + 3\cdot 5 & 0\cdot(-1) + 3\cdot(-2) \end{bmatrix}$$

$\parallel$

$$3\cdot A = \begin{bmatrix} 12 & -3 \\ 15 & -6 \end{bmatrix} = 3\begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix}$$

2.1.4

$$A = \begin{bmatrix} 9 & -1 & 3 \\ -8 & 7 & -6 \\ -4 & 1 & 8 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Regn ut

$A - 5I_3 \qquad (5I_3)A$

$$A - 5I_3 = \begin{bmatrix} 9 & -1 & 3 \\ -8 & 7 & -6 \\ -4 & 1 & 8 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -1 & 3 \\ -8 & 2 & -6 \\ -4 & 1 & 3 \end{bmatrix}$$

$$(5I_3) \cdot A = 5(I_3 \cdot A) = 5 \cdot A$$

$$5 \begin{bmatrix} 9 & -1 & 3 \\ -8 & 7 & -6 \\ -4 & 1 & 8 \end{bmatrix} = \begin{bmatrix} 45 & -5 & 15 \\ -40 & 35 & -30 \\ -20 & 5 & 40 \end{bmatrix}$$

2.1.12

La $A = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}$. Finn en 2x2-matrise

B slik at $A \cdot B$ er nullmatrisen

Bruk to forskjellige ikkenull kolonner for B.

$$\begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$3a - 6c = 0$

$3b - 6d = 0$

$\cancel{-a + 2c = 0}$

$-b + 2d = 0$

$a = 2c$

$b = 2d$

Eksempel, velg $c = 1$ og $d = 2$

Kunne valgt $c = 3$ og $d = -1$

For eksempel $B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 6 & -2 \\ 3 & -1 \end{bmatrix}$, annet eksempel.

9.2.3

$$A = \begin{pmatrix} 3 & -2 & 5 \\ 2 & 14 & 7 \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} -1 \\ 11 \end{pmatrix}$$

a) Regn ut, om mulig:

i) $A \cdot \vec{v} = \begin{pmatrix} 3 & -2 & 5 \\ 2 & 14 & 7 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 11 \end{pmatrix}$

Ikke definert.

ii) $A^T \cdot \vec{v} = \begin{pmatrix} 3 & 2 \\ -2 & 14 \\ 5 & 7 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 11 \end{pmatrix} = \begin{pmatrix} 3\cdot(-1) + 2\cdot 11 \\ (-2)\cdot(-1) + 14\cdot 11 \\ 5\cdot(-1) + 7\cdot 11 \end{pmatrix}$

$$= \begin{pmatrix} 19 \\ 156 \\ 72 \end{pmatrix}$$

iii) $AA^T + \vec{v}\vec{v}^T$

$$\underbrace{\begin{pmatrix} 3 & -2 & 5 \\ 2 & 14 & 7 \end{pmatrix} \cdot \begin{pmatrix} 3 & 2 \\ -2 & 14 \\ 5 & 7 \end{pmatrix}}_{\substack{2\times 3 \;\cdot\; 3\times 2 \\ 2\times 2}} + \underbrace{\begin{pmatrix} -1 \\ 11 \end{pmatrix} \cdot \begin{pmatrix} -1 & 11 \end{pmatrix}}_{\substack{2\times 1 \;\cdot\; 1\times 2 \\ \cancel{2}\; 2\times 2}}$$

$$\begin{pmatrix} 3\cdot 3 + (-2)\cdot(-2) + 5\cdot 5 & 3\cdot 2 - 2\cdot 14 + 5\cdot 7 \\ 2\cdot 3 - 2\cdot 14 + 7\cdot 5 & 2\cdot 2 + 14\cdot 14 + 7\cdot 7 \end{pmatrix} + \begin{pmatrix} (-1)\cdot(-1) & -1\cdot 11 \\ 11\cdot(-1) & 11\cdot 11 \end{pmatrix}$$

$$\begin{pmatrix} 38 & 13 \\ 13 & 249 \end{pmatrix} + \begin{pmatrix} 1 & -11 \\ -11 & 121 \end{pmatrix} = \begin{pmatrix} 39 & 2 \\ 2 & 370 \end{pmatrix}$$

b) Hvilken størrelse må B ha for at
$A \cdot B \cdot \vec{v}$ skal være definert.

$(2 \times 3) \cdot (3 \times 2) \cdot (2 \times 1)$

B må være en 3×2-matrise.

9.2.7

Regn ut $P \cdot Q$ og $Q \cdot P$ når

$$P = \begin{pmatrix} 1 & 0 \\ 3 & -2 \\ 1 & 1 \end{pmatrix} \qquad Q = \begin{pmatrix} 2 & 3 & 5 \\ 7 & 9 & -11 \end{pmatrix}$$

Er disse like?

$P \cdot Q$

$3 \times 2 \cdot 2 \times 3$

3×3

$Q \cdot P$

$2 \times 3 \cdot 3 \times 2$

2×2

$$P \cdot Q = \begin{pmatrix} 1 & 0 \\ 3 & -2 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 3 & 5 \\ 7 & 9 & -11 \end{pmatrix}$$

$$= \begin{pmatrix} 1\cdot 2 + 0\cdot 7 & 1\cdot 3 + 0\cdot 9 & 1\cdot 5 + 0\cdot(-11) \\ 3\cdot 2 + (-2)\cdot 7 & 3\cdot 3 + (-2)\cdot 9 & 3\cdot 5 + (-2)\cdot(-11) \\ 1\cdot 2 + 1\cdot 7 & 1\cdot 3 + 1\cdot 9 & 1\cdot 5 + 1\cdot(-11) \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 3 & 5 \\ -8 & -9 & 37 \\ 9 & 12 & -6 \end{pmatrix}$$

$$Q \cdot P = \begin{pmatrix} 2 & 3 & 5 \\ 7 & 9 & -11 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 3 & -2 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2\cdot 1 + 3\cdot 3 + 5\cdot 1 & 2\cdot 0 + 3\cdot(-2) + 5\cdot 1 \\ 7\cdot 1 + 9\cdot 3 + (-11)\cdot 1 & 7\cdot 0 + 9\cdot(-2) + (-11)\cdot 1 \end{pmatrix}$$

$$= \begin{pmatrix} 16 & -1 \\ 23 & -29 \end{pmatrix}$$

$$Q \cdot P = \begin{pmatrix} 16 & -1 \\ 23 & -29 \end{pmatrix}.$$

$$\left(\begin{array}{cc|cc} 16 & -1 & 1 & 0 \\ 23 & -29 & 0 & 1 \end{array}\right) \overset{\frac{1}{16}\cdot \mathrm{I}}{\sim} \left(\begin{array}{cc|cc} 1 & -\frac{1}{16} & \frac{1}{16} & 0 \\ 23 & -29 & 0 & 1 \end{array}\right)$$

$$\overset{\mathrm{II}-23\cdot \mathrm{I}}{\sim} \left(\begin{array}{cc|cc} 1 & -\frac{1}{16} & \frac{1}{16} & 0 \\ 0 & \frac{23}{16}-29 & -\frac{23}{16} & 1 \end{array}\right) = \left(\begin{array}{cc|cc} 1 & -\frac{1}{16} & \frac{1}{16} & 0 \\ 0 & -\frac{441}{16} & -\frac{23}{16} & 1 \end{array}\right)$$

$$\overset{-\frac{16}{441}\cdot \mathrm{II}}{\sim} \left(\begin{array}{cc|cc} 1 & -\frac{1}{16} & \frac{1}{16} & 0 \\ 0 & 1 & \frac{23}{441} & -\frac{16}{441} \end{array}\right) \overset{\mathrm{I}+\frac{1}{16}\cdot \mathrm{II}}{\sim} \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{16}+\frac{1}{16}\cdot\frac{23}{441} & -\frac{1}{16}\cdot\frac{16}{441} \\ 0 & 1 & \frac{23}{441} & -\frac{16}{441} \end{array}\right)$$

$$\frac{1}{16} + \frac{1}{16} \cdot \frac{23}{441}$$

$$\frac{1}{16} \cdot \frac{-16}{441}$$

$$= \left(\begin{array}{cc|cc} 1 & 0 & \frac{464}{7056} & -\frac{1}{441} \\ 0 & 1 & \frac{23}{441} & -\frac{16}{441} \end{array}\right)$$

$$(Q \cdot P)^{-1} = \begin{pmatrix} \frac{29}{441} & -\frac{1}{441} \\ \frac{23}{441} & -\frac{16}{441} \end{pmatrix}$$

$$Q \cdot P = \begin{pmatrix} 16 & -1 \\ 23 & -29 \end{pmatrix}$$

$$(Q \cdot P)^{-1} = \frac{1}{\det(Q \cdot P)} \cdot \begin{pmatrix} -29 & 1 \\ -23 & 16 \end{pmatrix}$$

$$\det Q \cdot P = 16 \cdot (-29) - (-1) \cdot 23 = -441$$

$$(Q \cdot P)^{-1} = \frac{1}{-441} \cdot \begin{pmatrix} -29 & 1 \\ -23 & 16 \end{pmatrix} = \begin{pmatrix} \frac{29}{441} & -\frac{1}{441} \\ \frac{23}{441} & -\frac{16}{441} \end{pmatrix}$$

$$P\cdot Q = \begin{pmatrix} 2 & 3 & 5 \\ -8 & -9 & 37 \\ 9 & 12 & -6 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 2 & 3 & 5 & 1 & 0 & 0 \\ -8 & -9 & 37 & 0 & 1 & 0 \\ 9 & 12 & -6 & 0 & 0 & 1 \end{array}\right) \overset{III+II}{\sim} \begin{pmatrix} 2 & 3 & 5 & 1 & 0 & 0 \\ -8 & -9 & 37 & 0 & 1 & 0 \\ 1 & 3 & 31 & 0 & 1 & 1 \end{pmatrix}$$

$$\overset{I \leftrightarrow III}{\sim} \begin{pmatrix} 1 & 3 & 31 & 0 & 1 & 1 \\ -8 & -9 & 37 & 0 & 1 & 0 \\ 2 & 3 & 5 & 1 & 0 & 0 \end{pmatrix} \underset{III-2\cdot I}{\overset{II+8\cdot I}{\sim}} \begin{pmatrix} 1 & 3 & 31 & 0 & 1 & 1 \\ 0 & 15 & 285 & 0 & 9 & 8 \\ 0 & -3 & -57 & 1 & -2 & -2 \end{pmatrix}$$

$$\overset{\frac{1}{15}\cdot II}{\sim} \begin{pmatrix} 1 & 3 & 31 & 0 & 1 & 1 \\ 0 & 1 & 19 & 0 & \frac{3}{5} & \frac{8}{15} \\ 0 & -3 & -57 & 1 & -2 & -2 \end{pmatrix} \overset{III+3\cdot II}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 31 & 0 & 1 & 1 \\ 0 & 1 & 19 & 0 & \frac{3}{5} & \frac{8}{15} \\ 0 & 0 & 0 & 1 & -\frac{1}{5} & -\frac{8}{5} \end{array}\right)$$

$$-2+3\cdot\frac{3}{5} = \frac{9}{5} - \frac{10}{5} = -\frac{1}{5}$$

$$-2+\frac{3\cdot 8}{15} = \frac{24}{15} - \frac{30}{15} = \frac{-6}{15}$$

$P\cdot Q$ har ingen invers.

2.2.1

Finn inversen til $\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 8 & 6 & 1 & 0 \\ 5 & 4 & 0 & 1 \end{array}\right] \overset{\frac{1}{8}\cdot I}{\sim} \begin{bmatrix} 1 & \frac{3}{4} & \frac{1}{8} & 0 \\ 5 & 4 & 0 & 1 \end{bmatrix}$$

$$\overset{II-5\cdot I}{\sim} \begin{bmatrix} 1 & \frac{3}{4} & \frac{1}{8} & 0 \\ 0 & \frac{1}{4} & -\frac{5}{8} & 1 \end{bmatrix} \overset{4\cdot II}{\sim} \begin{bmatrix} 1 & \frac{3}{4} & \frac{1}{8} & 0 \\ 0 & 1 & -\frac{5}{2} & 4 \end{bmatrix}$$

$$\overset{I-\frac{3}{4}II}{\sim} \left[\begin{array}{cc|cc} 1 & 0 & 2 & -3 \\ 0 & 1 & -\frac{5}{2} & 4 \end{array}\right]$$

Inversen er $\begin{bmatrix} 2 & -3 \\ -\frac{5}{2} & 4 \end{bmatrix}$

$$\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}^{-1} = \frac{1}{2}\begin{bmatrix} 4 & -6 \\ -5 & 8 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -\frac{5}{2} & 4 \end{bmatrix}$$

2.2.5. Bruk inversen til å løse

$$8x_1 + 6x_2 = 2$$
$$5x_1 + 4x_2 = -1$$

$$\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \\ -\frac{5}{2} & 4 \end{bmatrix} \begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -\frac{5}{2} & 4 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ -9 \end{bmatrix}$$

$x_1 = 7$

$x_2 = -9.$