9.1.10

Finn t slik at konsistent:

$$\begin{pmatrix} 2 & t & 7 \\ 1 & 8 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 8 & -2 \\ 2 & t & 7 \end{pmatrix}$$

$$\overset{II-2\cdot I}{\sim} \begin{pmatrix} 1 & 8 & -2 \\ 0 & \boxed{t-16} & 11 \end{pmatrix}$$

$x + 8y = -2$

$(t-16)y = 11$

Dette kan løses så lenge $t - 16 \neq 0$

$\Rightarrow t \neq 16$

9.1.11) Når er det ingen, én eller vendelig mange løsn?

a) $x + (t+2)y = t$

$tx + 3y = 1$

$$\begin{pmatrix} 1 & t+2 & t \\ t & 3 & 1 \end{pmatrix} \overset{II-t\cdot I}{\sim} \begin{pmatrix} 1 & t+2 & t \\ 0 & 3-t(t+2) & 1-t\cdot t \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & t+2 & t \\ 0 & \boxed{-t^2-2t+3} & 1-t^2 \end{pmatrix}$$

Når er $-t^2 - 2t + 3 = 0$? $t=1$ og $t=-3$

Ha én løsning når $t \neq 1$ og $t \neq -3$

$t=1$

$$\begin{pmatrix} 1 & 3 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$ Uendelig mange løsningen

$t=-3$

$$\begin{pmatrix} 1 & -1 & -3 \\ 0 & 0 & -8 \end{pmatrix}$$ Ingen løsning.

$-t^2-2t+3=0$

$t^2+2t-3=0$

$a \cdot b = -3$

$a+b=-2$

$a=1$

$b=-3$

---

b)

$3x + (t+1)y = 3$

$tx + 4y = 3$

$$\begin{pmatrix} 3 & t+1 & 3 \\ t & 4 & 3 \end{pmatrix} \overset{\frac{1}{3}\cdot I}{\sim} \begin{pmatrix} 1 & \frac{t+1}{3} & 1 \\ t & 4 & 3 \end{pmatrix}$$

$$\overset{II - t\cdot I}{\sim} \begin{pmatrix} 1 & \frac{t+1}{3} & 1 \\ 0 & 4-t\left(\frac{t+1}{3}\right) & 3-t \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{t+1}{3} & 1 \\ 0 & \boxed{-\frac{1}{3}t^2-\frac{1}{3}t+4} & 3-t \end{pmatrix}$$

Vil løse $-\frac{1}{3}t^2 - \frac{1}{3}t + 4 = 0$ $\quad 1 \cdot -3$

$t^2 + t - 12 = 0$ $\quad \boxed{t=3 \quad t=-4}$

Vi har én løsning når $t \neq 3$ og $t \neq -4$

$t=3$
$$\begin{pmatrix} 1 & \frac{4}{3} & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Vendelig mange løsninger

$t=-4$
$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & 7 \end{pmatrix}$$

Ingen løsning.

c)

$$(t-2)x + 4y = -12$$
$$2x + ty = 6$$

$$\begin{pmatrix} t-2 & 4 & -12 \\ 2 & t & 6 \end{pmatrix} \sim \begin{pmatrix} 2 & t & 6 \\ t-2 & 4 & -12 \end{pmatrix}$$

$$\overset{\frac{1}{2}\cdot I}{\sim} \begin{pmatrix} 1 & \frac{t}{2} & 3 \\ t-2 & 4 & -12 \end{pmatrix} \overset{II-(t-2)\cdot I}{\sim} \begin{pmatrix} 1 & \frac{t}{2} & 3 \\ 0 & 4-(t-2)\frac{t}{2} & -12-(t-2)\cdot 3 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & \frac{t}{2} & 3 \\ 0 & -\frac{t^2}{2}+t+4 & -3t-6 \end{pmatrix}$$

$$-\frac{t^2}{2}+t+4 = 0 \quad | \cdot -2$$
$$t^2-2t-8=0 \qquad t=-2,\ t=4$$

Én løsning når $t \neq -2,\ t \neq 4$

$t=-2$

$$\begin{pmatrix} 1 & -1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

Uendelig mange løsninger

$t=4$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & -18 \end{pmatrix}$$

Ingen løsning.

$$\begin{pmatrix} t-2 & 4 & -12 \\ 2 & t & 6 \end{pmatrix} \overset{\frac{1}{t-2}\cdot I}{\sim} \begin{pmatrix} 1 & \frac{4}{t-2} & -\frac{12}{t-2} \\ 2 & t & 6 \end{pmatrix}$$

$$\overset{II-2\cdot I}{\sim} \begin{pmatrix} 1 & \frac{4}{t-2} & -\frac{12}{t-2} \\ 0 & t-\frac{8}{t-2} & 6+\frac{24}{t-2} \end{pmatrix}$$

$$t-\frac{8}{t-2}=0 \qquad t=\frac{8}{t-2} \quad \Big| \cdot t-2$$

$$t^2-2t=8 \qquad t^2-2t-8=0$$

Må også sjekke $t=2$

$$\begin{pmatrix} 0 & 4 & -12 \\ 2 & 2 & 6 \end{pmatrix} \sim \begin{pmatrix} 2 & 2 & 6 \\ 0 & 4 & -12 \end{pmatrix}$$

$$\frac{x^2-x}{x}=\frac{0}{x} \Rightarrow x-1=0$$

$$x=1$$

Mister $x=0$-løsningen.

$$x(x-1)=0 \Rightarrow \begin{matrix} x=0 \\ x-1=0 \end{matrix}$$

9.1.12

a) $5x - 2y + tz = 4$

$6x + 5y + z = 1$

$5x + 4y + z = 2$

$$\begin{pmatrix} 5 & -2 & t & 4 \\ 6 & 5 & 1 & 1 \\ 5 & 4 & 1 & 2 \end{pmatrix} \overset{II-I}{\sim} \begin{pmatrix} 5 & -2 & t & 4 \\ 1 & 7 & 1-t & -3 \\ 5 & 4 & 1 & 2 \end{pmatrix}$$

$$\overset{II \Leftrightarrow I}{\sim} \begin{pmatrix} 1 & 7 & 1-t & -3 \\ 5 & -2 & t & 4 \\ 5 & 4 & 1 & 2 \end{pmatrix} \underset{III-5\cdot I}{\overset{II-5\cdot I}{\sim}} \begin{pmatrix} 1 & 7 & 1-t & -3 \\ 0 & -37 & 6t-5 & 19 \\ 0 & -31 & 5t-4 & 17 \end{pmatrix}$$

$$\overset{-\frac{1}{37}\cdot II}{\sim} \begin{pmatrix} 1 & 7 & 1-t & -3 \\ 0 & 1 & \frac{5-6t}{37} & \frac{-19}{37} \\ 0 & -31 & 5t-4 & 17 \end{pmatrix} \overset{III+31\cdot I}{\sim} \begin{pmatrix} 1 & 7 & 1-t & -3 \\ 0 & 1 & \frac{5-6t}{37} & -\frac{19}{37} \\ 0 & 0 & \boxed{5t-4+31\frac{5-6t}{37}} & 17-31\cdot\frac{19}{37} \end{pmatrix}$$

$$5t - 4 + 31\cdot\frac{5-6t}{37} = 0 \quad | \cdot 37$$

$$5\cdot 37t - 4\cdot 37 + 31\cdot 5 - 31\cdot 6t = 0$$

$$(5\cdot 37 - 6\cdot 31)t + 31\cdot 5 - 4\cdot 37 = 0$$

$$-t + 7 = 0$$

$$7 = t$$

$t \neq 7$: én løsning.

$t = 7$ $\qquad 0 = 1{,}08 \qquad$ Ingen løsning.

3

c)

$$
\begin{aligned}
3x + 3ty + 30z &= 45 \\
x + ty + 5z &= 0 \\
-4x + 8y &= s
\end{aligned}
$$

$$
\begin{pmatrix} 3 & 3t & 30 & 45 \\ 1 & t & 5 & 0 \\ -4 & 8 & 0 & s \end{pmatrix}
\underset{\sim}{I \leftrightarrow II}
$$

$$
\underset{\sim}{I \leftrightarrow II}
\begin{pmatrix} 1 & t & 5 & 0 \\ 3 & 3t & 30 & 45 \\ -4 & 8 & 0 & s \end{pmatrix}
$$

$$
\underset{III + 4 \cdot I}{\overset{II - 3 \cdot I}{\sim}}
\begin{pmatrix} 1 & t & 5 & 0 \\ 0 & 0 & 15 & 45 \\ 0 & 8+4t & 20 & s \end{pmatrix}
$$

$$
\underset{\sim}{III \leftrightarrow II}
\begin{pmatrix} 1 & t & 5 & 0 \\ 0 & \boxed{8+4t} & 20 & s \\ 0 & 0 & 15 & 45 \end{pmatrix}
$$

$$
\underset{\sim}{\frac{1}{15} \cdot III}
\begin{pmatrix} 1 & t & 5 & 0 \\ 0 & 8+4t & 20 & s \\ 0 & 0 & 1 & 3 \end{pmatrix}
$$

$8 + 4t = 0 \qquad t = -2$

Når $t \neq -2$, én løsning.

$t = -2$

$$
\begin{pmatrix} 1 & -2 & 5 & 0 \\ 0 & 0 & 20 & s \\ 0 & 0 & 1 & 3 \end{pmatrix}
$$

$$
\underset{\sim}{II \leftrightarrow III}
\begin{pmatrix} 1 & -2 & 5 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 20 & s \end{pmatrix}
$$

$$
\underset{\sim}{III - 20 II}
\begin{pmatrix} 1 & -2 & 5 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & s-60 \end{pmatrix}
$$

$s = 60$: Uendelig mange

$s \neq 60$: Ingen løsning

1.2.13 LA-boka:

Finn løsning til systemet som har matrise

$$\begin{pmatrix} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\underset{\sim}{\mathrm{I}+\mathrm{III}} \begin{pmatrix} 1 & -3 & 0 & 0 & 9 & 2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\underset{\sim}{\mathrm{I}+3\cdot\mathrm{II}} \begin{pmatrix} 1 & 0 & 0 & 0 & -3 & 5 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} x_1 - 3x_5 &= 5 \\ x_2 - 4x_5 &= 1 \\ x_4 + 9x_5 &= 4 \end{aligned}$$

$$\begin{aligned} x_1 &= 5 + 3x_5 \\ x_2 &= 1 + 4x_5 \\ x_3 &= x_3 \\ x_4 &= 4 - 9x_5 \\ x_5 &= x_5 \end{aligned}$$

$$(5,1,0,4,0) + x_3(0,0,1,0,0) + x_5(3,4,0,-9,1)$$

9.1.6 c)

$$x - y - 2z + w = 0$$
$$-2x + 2y + 4z + w = 6$$
$$x \quad - 3z + w = 1$$

$$\begin{pmatrix} 1 & -1 & -2 & 1 & 0 \\ -2 & 2 & 4 & 1 & 6 \\ 1 & 0 & -3 & 1 & 1 \end{pmatrix}$$

$$\begin{array}{c} \\ \mathrm{II}+2\mathrm{I} \\ \sim \\ \mathrm{III}-\mathrm{I} \end{array} \begin{pmatrix} 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 3 & 6 \\ 0 & 1 & -1 & 0 & 1 \end{pmatrix}$$

$$\mathrm{II} \leftrightarrow \mathrm{III} \sim \begin{pmatrix} 1 & -1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 3 & 6 \end{pmatrix}$$

$$\tfrac{1}{3}\cdot\mathrm{III} \sim \begin{pmatrix} 1 & -1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

$$\mathrm{I}-\mathrm{III} \sim \begin{pmatrix} 1 & -1 & -2 & 0 & -2 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix} \mathrm{I}+\mathrm{II} \sim \begin{pmatrix} 1 & 0 & -3 & 0 & -1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

$$x - 3z = -1$$
$$y - z = 1$$
$$w = 2$$

$$\uparrow$$

$$x = -1 + 3z$$
$$y = 1 + z$$
$$w = 2$$
$$z = z$$

$$\uparrow$$

$$(-1, 1, 0, 2) + z(3, 1, 1, 0)$$

1.1.19 i L.A.

$$\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix} \overset{II-3\cdot I}{\sim} \begin{bmatrix} 1 & h & 4 \\ 0 & 6-3h & -4 \end{bmatrix}$$

Når er $6-3h=0 \Rightarrow h=2$

$h \neq 2$, nøyaktig én løsning

$h=2$, ingen løsning.

1.1.20

$$\begin{bmatrix} 1 & h & -3 \\ -2 & 4 & -6 \end{bmatrix} \overset{II+2\cdot I}{\sim} \begin{bmatrix} 1 & h & -3 \\ 0 & \boxed{4+2h} & -12 \end{bmatrix}$$

$4+2h=0 \Rightarrow h=-2$

$h \neq -2$, én løsning

$h=-2$, ingen løsning.

1.1.21

$$\begin{bmatrix} 1 & 3 & -2 \\ -4 & h & 8 \end{bmatrix} \overset{II+4\cdot I}{\sim} \begin{bmatrix} 1 & 3 & -2 \\ 0 & h+12 & 0 \end{bmatrix}$$

$h=-12 \Rightarrow$ Vendelig mange løsn.

$h \neq -12 \Rightarrow$ Nøyaktig én løsning.

1.1.22

$$\begin{bmatrix} 2 & -3 & 4 \\ -6 & 9 & 5 \end{bmatrix} \overset{\frac{1}{2}\cdot I}{\sim} \begin{bmatrix} 1 & -\frac{3}{2} & \frac{4}{2} \\ -6 & 9 & 5 \end{bmatrix}$$

$$\overset{II+6\cdot I}{\sim} \begin{bmatrix} 1 & -\frac{3}{2} & \frac{4}{2} \\ 0 & 0 & 5+3h \end{bmatrix}$$

$$5+3h=0 \Rightarrow h=-\frac{5}{3}$$

$h=-\frac{5}{3} \Rightarrow$ Uendelig mange løsninger

$h \neq -\frac{5}{3} \Rightarrow$ Ingen løsning.