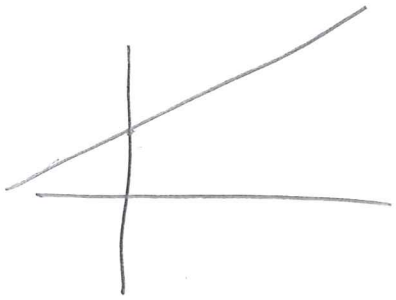
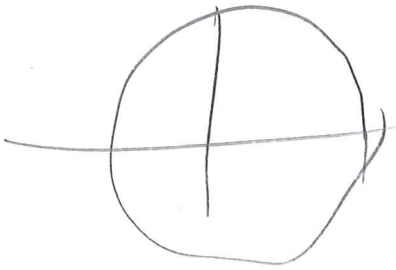
Lineare likningssystem

$2x + 3y = 5$



$x^2 + y^2 = 4$

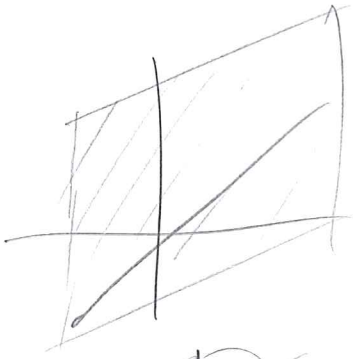


$a \cdot x + b \cdot y = c$

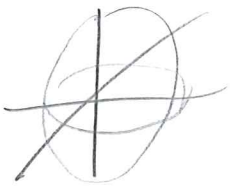
Lineær likning

$3x - 2y + 5z = 12$

lineær likning



$x^2 + y^2 + z^2 = 4$



$3x - 5y + 4z - 3w = 7$

Et likningssystem består av flere lineære likninger.

$$2x - y = 5$$
$$x + 2y = 0$$

En løsning er x- og y-verdier slikat begge stemmer: Her: $x = 2$ og $y = -1$ er en løsning.

$$2 \cdot 2 - (-1) = 5$$
$$2 + 2(-1) = 0$$

Innsettingsmetoden:

$$x = -2y$$
$$2(-2y) - y = 5$$
$$-5y = 5$$
$$y = -1$$

$$x = -2 \cdot (-1)$$
$$x = 2$$

Addisjonsmetoden:

(I) $2x - y = 5$
(II) $x + 2y = 0$

(I) $- 2 \cdot$ (II)

$$2x - y - 2(x + 2y) = 5 - 2 \cdot 0$$
$$-5y = 5$$

$$-5y = 5$$
$$x + 2y = 0$$

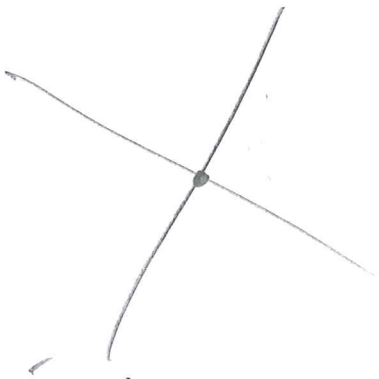
$x + 2y = 0$
$-5y = 5$

Løsninger:

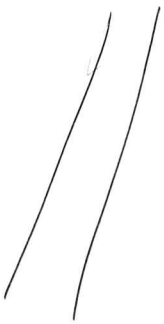
Er likningssettet konsistent? Har det minst én løsning?

Er løsningen unik? Finnes nøyaktig én løsning?

Tre muligheter for 2 likninger, 2 ukjente:



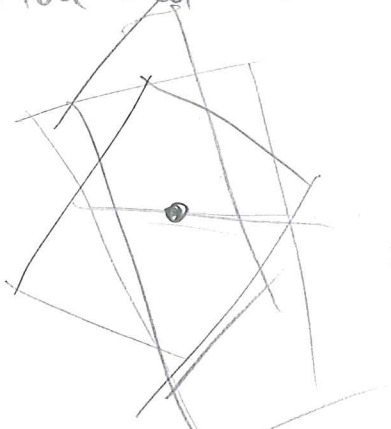
Én løsning



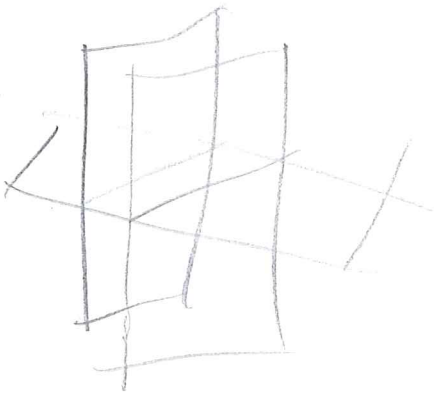
Null løsninger

Uendelig mange løsninger.

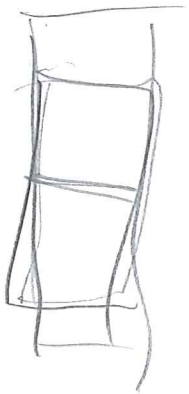
Hva med 3 likninger, 3 ukjente?



En løsning

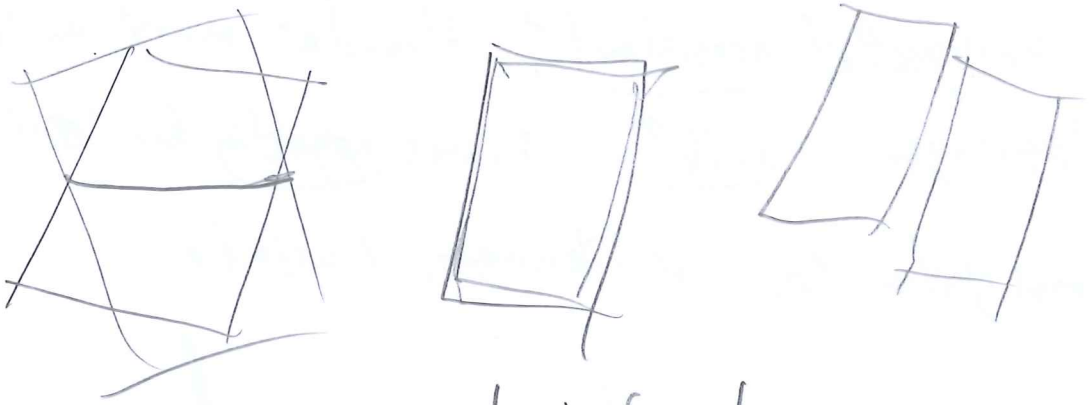


Ingen løsning



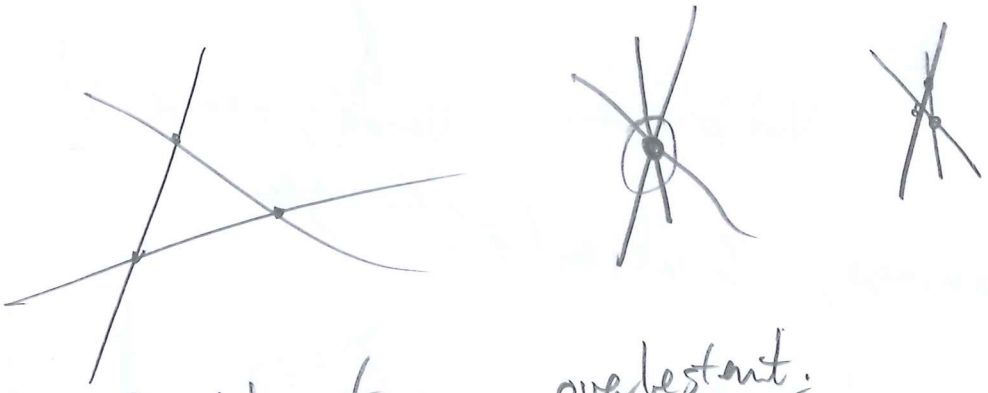
Uendelig mange

Hva med 2 likninger, 3 ukjente?



Problemet er underbestemt

Hva med 3 likninger, 2 ukjente?



Problemet er overbestemt.

Eks:

$$\begin{matrix} \text{I} & x+y=1 \\ \text{II} & 2x+2y=3 \end{matrix} \Big\} \text{ Ingen løsning.}$$

$2\cdot\text{I}$: $2x+2y=2$

(I) $x - 2y = 8$

(II) $5x + 3y = 1$

(II) -5 (I) $\Rightarrow$

$x - 2y = 8$

$13y = -39$

(II) $\cdot \frac{1}{13}$ $\Rightarrow$

$x - 2y = 8$

$y = -3$

Setter inn $\Rightarrow$ $x - 2(-3) = 8$ $\quad x = 2$

$y = -3$

I $2x + 6y = 4$

II $3x + 9y = 11$

$\frac{1}{2} \cdot$ (I) $\Rightarrow$

$x + 3y = 2$

$3x + 9y = 11$

II $- 3 \cdot$ I $\Rightarrow$

$x + 3y = 2$

$0x + 0y = 5$

$x + 3y = 2$

$0 = 5$

Ingen løsning!

I $3x + 9y = 6$

II $x + 3y = 2$ $\Rightarrow$

$x + 3y = 2$

$3x + 9y = 6$

II $- 3$I $\Rightarrow$

$x + 3y = 2$

$0x + 0y = 0$ $\Rightarrow$

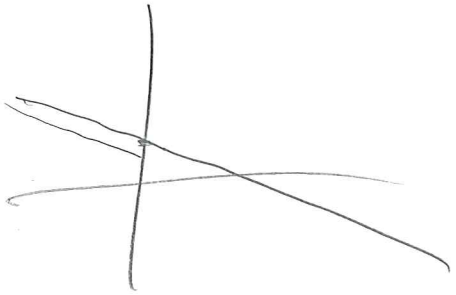
$x + 3y = 2$

$0 = 0$

Uendelig mange løsninger.

$3y = 2 - x$

$y = -\frac{1}{3}x + \frac{2}{3}$



Tre regler for addisjonsmetoden:

1. Vi kan bytte plass på likninger.
2. Gange en likning med et tall $(\neq 0)$
3. Ene likninga pluss et tall ganget den andre likninga.

$$\begin{aligned} \text{I}\quad & x+2y+z=4 \\ \text{II}\quad & 3x+8y+7z=20 \\ \text{III}\quad & 2x+7y+9z=23 \end{aligned}$$

$\text{II}-3\cdot\text{I}$, $\text{III}-2\cdot\text{I}$ $\Rightarrow$

$$\begin{aligned} & x+2y+z=4 \\ & 2y+4z=8 \\ & 3y+7z=15 \end{aligned}$$

$\frac{1}{2}\cdot\text{II}$ $\Rightarrow$

$$\begin{aligned} & x+2y+z=4 \\ & y+2z=4 \\ & 3y+7z=15 \end{aligned}$$

$\text{III}-3\cdot\text{II}$ $\Rightarrow$

$$\begin{aligned} & x+2y+z=4 \\ & y+2z=4 \\ & z=3 \end{aligned}$$

Setter inn $\Rightarrow$

$$\begin{aligned} & x+2y+3=4 \\ & y+6=4 \\ & z=3 \end{aligned} \quad \Rightarrow \quad \begin{aligned} & x+2y=1 \\ & y=-2 \\ & z=3 \end{aligned}$$

Setter inn $\Rightarrow$

$$\begin{aligned} & x+2(-2)=1 \\ & y=-2 \\ & z=3 \end{aligned} \quad \Rightarrow \quad \begin{aligned} & x=5 \\ & y=-2 \\ & z=3 \end{aligned}$$

Kunne også:

$$\begin{aligned} x+2y+z &= 4 \quad \text{(I)}-\text{(III)} \\ y+2z &= 4 \quad \Rightarrow \\ z &= 3 \end{aligned}$$

$$\begin{aligned} x+2y &= 1 \\ y &= -2 \\ z &= 3 \end{aligned}$$

(II) − 2·(III)

(I) − 2(II) $\Rightarrow$

$$\begin{aligned} x &= 5 \\ y &= -2 \\ z &= 3 \end{aligned}$$

$$\begin{aligned} 2x+3y-z &= 5 \\ x-2y+z &= 1 \\ 4x+6y-2z &= 7 \end{aligned}$$

$$\begin{aligned} x-2y+z &= 1 \\ 2x+3y-z &= 5 \\ 4x+6y-2z &= 7 \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} x-2y+z &= 1 \\ 7y-3z &= 3 \\ 14y-6z &= 3 \end{aligned} \Rightarrow \begin{aligned} x-2y+z &= 1 \\ y-\tfrac{3}{7}z &= \tfrac{3}{7} \\ 14y-6z &= 3 \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} x-2y+z &= 1 \\ y-\tfrac{3}{7}z &= \tfrac{3}{7} \\ 0 &= -3 \end{aligned}$$

Ingen løsning